POLYCHROMATIC HILBERT–DIAGNOSTICS OF VORTICAL STRUCTURES AND FREE–CONVECTIVE JETS

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ABSTRACT: The method polychromatic Hilbert–filtration is used for visualization of fields of optical phase density in flows. Color flow visualization is carried out at use the quadrant Hilbert–filter and a source of white light. In case of use of a white light linear source or a polychromatic source on various lengths of waves the colored one–dimensional Hilbert transformation and one–dimensional Foucault–Hilbert transformation with various weight λ–factors is carried out. Thus sensitivity increases due to color contrast and there is an opportunity of definition of phase gradients in a wide dynamic range. Hilbert transformation redistributes energy of the optical signal in high–frequency area of a spatial Fourier–spectrum. Hilbert–filtration in comparison with usual schlieren–visualization achieves the raised sensitivity due to color contrast. Besides color gradients in the Hilbert–filtered signal carry an information on gradients of the refractive index in the structure of the optical phase density. Various applications of this method in the investigation of vortical structures and jets are discussed.

INTRODICTION

The methods of diagnostics based on the Hilbert–filtration of light fields, have found fruitful application in experimental hydrodynamics and gas dynamics [1–3]. First of all, it concerns the problems connected with necessity of visualization of optical phase density in the flow investigated. Hilbert–transformation of an optical signal is realized usually with phase filters having a various configuration [2–4]. Intensively methods of isotropic transformation of Hilbert in optics develop now. The polychromatic Hilbert–visualization opens up a whole range of new opportunities due to color contrast. Features of application of the Hilbert–optics in some tasks of experimental hydrodynamics and gas dynamics are discussed in this report . As an example the turbulent current of gas known as a vortical ring is investigated. Other object of researches is evolution of floating jets above a linear source of heat in strongly viscous liquid.

1. OPTICAL HILBERT AND FOUCAULT–HILBERT'S TRANSFORMATION

As is known, one–dimensional Hilbert transformation in the coordinate space is described by convolution of a signal $s(x)$ with function $\frac{1}{\pi x}$. In the frequency space it corresponds to product of a Fourier–spectrum of the signal on function $H(K_x) = -i\,\mathrm{sgn}\,K_x$. Here K_x – a *x*-component of spatial frequency, $\mathrm{sgn}\,K_x$ – sign function, $\frac{1}{\pi x} \leftrightarrow -i\,\mathrm{sgn}\,K_x$. Function $H(K_x)$ is coherently–transfer function (CTF) of the corresponding filter.

On fig. 1 a sketch of the confocal optical schem carrying out Hilbert–filtration of optical signals in systems of flows diagnostics is shown. A source 0 and an objective 1 form the probing light field extending through the fluid under study. The light field $s(x, y)$ disturbed by optical nonuniformities is transformed by an objective 2 to the spatially–frequency spectrum $s(K_x, K_y)$ localized in the Fourier–plane (K_x, K_y) which is an image plane for the light source 0. Here $K_x = \frac{k}{f} x_f$, $K_y = \frac{k}{f} y_f$, x_f and y_f – coordinates in the frequency plane. In the frequency plane the filter with corresponding coherently–transfer function $H(K_x, K_y)$ is located. Directly behind the filter a Fourier–spectrum of the filtered signal $s(K_x, K_y)H(K_x, K_y)$ is formed. The objective 3 carries out the Fourier–transformation and restores the filtered optical signal.

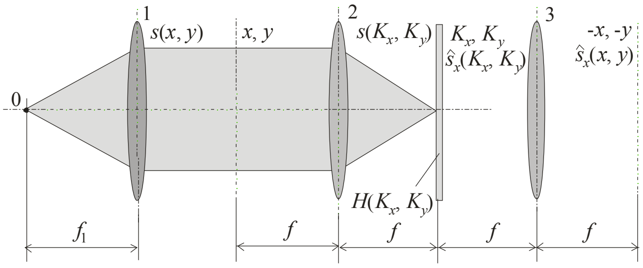


Fig. 1. The Hilbert–visualizater scheme

The optical Hilbert-transformation has similar with differentiation of a spatial signal. However, unlike operation of differentiation, Hilbert's transformation has not local character with conservation of energy which in Hilbert–conjugated signal is redistributed from low–frequency area to high–frequency area. It is an important advantage Hilbert–filtration in comparison with spatial differentiation. Optical Hilbert–transformation is carried out visualization of phase density of the fluid under investigation.

Let's direct attention to the quadrant phase filter. Generally its coherent transfer function is described by expression

$$H(K_x,K_y)=\left[e^{i\varphi}\sigma(K_x)+e^{-i\varphi}\sigma(-K_x)\right]\sigma(K_y)+\left[e^{-i\varphi}\sigma(K_x)+e^{i\varphi}\sigma(-K_x)\right]\sigma(-K_y)=$$

$$=\cos\varphi+i\sin\varphi\,\mathrm{sgn}\,K_x\,\mathrm{sgn}\,K_y\,. \qquad (1)$$

When the light source has point or cross-shaped form the filter with CTF (1) carries out Foucault–Hilbert's two–dimensional filtration with weight factors $\cos\varphi$ and $\sin\varphi$ for an initial signal and its Hilbert–image. Weight factors depend on a wave length of the source. This dependence influences on coloring and color contrast of the filtered image. For optical signals on waves lengths, satisfying to a condition $\cos\varphi=0$, $\sin\varphi=\pm1$, two–dimensional Hilbert transformation is carried out to within a sign:

$$s(K_x,K_y)=H(K_x,K_y)=s(K_x,K_y)\mathrm{sgn}\,K_x\,\mathrm{sgn}\,K_y\,. \qquad (2)$$

At use of the linear light source focused on one spatially–frequency axes, the filter (2) carries out the one–dimensional Hilbert–transformation, and the filter (1) performs one–dimensional Foucault–Hilbert transformation. The isotropic Hilbert- transformation can be received, for example, with application of the filter whose coherent transfer function in polar system of coordinates looks like [4, 5]

$$H(w,\theta)=e^{\pm i\theta}\,, \qquad (3)$$

where w – polar radius, θ – a polar angle.

As shown in [2, 4], the isotropic Hilbert–transformation of any optical signals is reduced to transformation axisymmetric components of the expansion of this signal in the series on polar harmonics, and restoration of the filtered signal in the coordinate space by generalized Hankel transformation of corresponding orders. Thus phase and amplitude filters with polar carrier on various angular frequencies can be used for Hilbert's isotropic transformation.

In case of use of a white light linear source or a polychromatic source on various lengths of waves the colored one–dimensional Hilbert transformation and one–dimensional Foucault–Hilbert transformation with various weight λ–factors is carried out. Thus sensitivity increases due to color contrast and there is an opportunity of definition of phase gradients in a wide dynamic range. We shall consider some examples of the practical application Hilbert–diagnostics.

2. HILBERT–VISUALIZATION OF VORTICAL RINGS

Vortical rings are object of researches already over 100 years since Helmholtz works. The history of these researches and the basic scientific results are present in many papers and monographies [7–8]. The most popular generator of the colored vortical rings represents the filled by a smoke chamber with an aperture. One of wall of the chamber is exposed to pulse loading. Whirlwinds which arise outside of the chamber, have been investigated and described. However experimental researches of the dynamic processes occuring inside of the chamber of the vortical rings generator practically are absent. Exception is the work [6] which for the first time reported about vortical rings induced by pressure jump on the aperture and moving inside of the chamber in a direction, opposite to movement of the external vortical rings. Difficulty of experimental detection of vortical rings inside of the chamber has been connected by that smoke blanketing camouflaged existence of the vortical structures, and weak indignations of optical density demanded application of high-sensitivity optical methods for visualisation. Therefore for detection of internal vortical structures Hilbert-optics methods have been used.

Evolution of the vortical rings induced by pressure jump on an aperture in a wall of the gas chamber whose construction is described in [6] was investigated. The pressure jump is created by means of the electrodynamic loudspeaker operated by the COMPUTER. The sizes of the chamber 0,19´0,19´0,38 м2. In the front wall of the chamber there is an aperture in a diameter of 20 mm, thickness of a wall of 3 mm. The back wall of the chamber is formed by the diffuser of an electrodynamic loudspeaker. High-quality optical windows are inserted into lateral walls for supervision of vortical structures inside of the chamber. The electrodynamic loudspeaker was controlled by electric impulses with desired polarity, amplitude and on-of time ratio. Visualisation of vortical structures was carried out by methods of Hilbert–filtration of phase disturbations of the light field caused by optical density indignations at occurrence and evolution of vortical rings inside and out of the chamber.

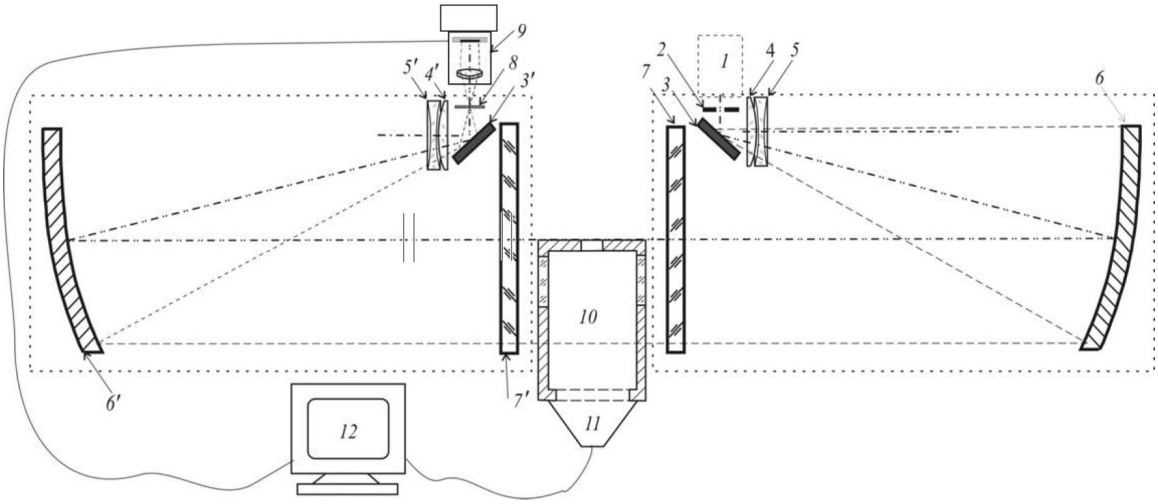
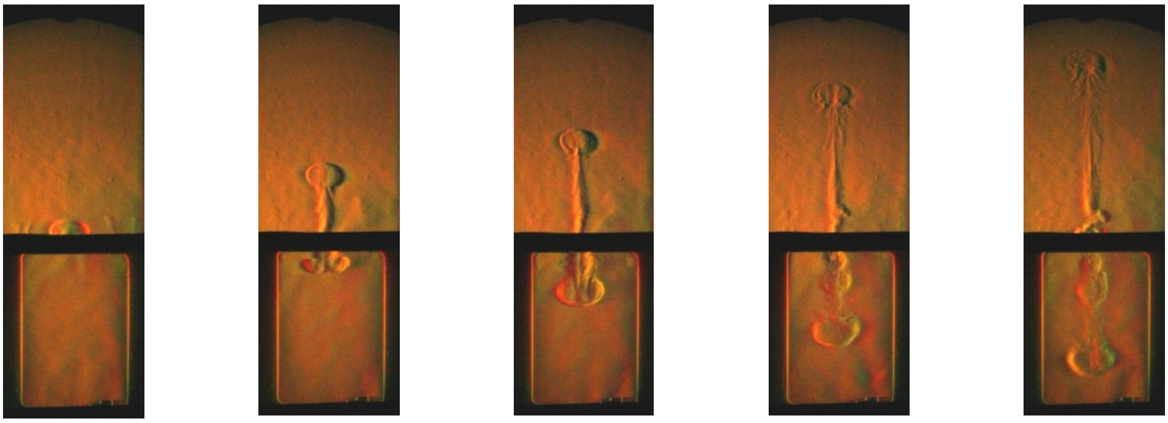


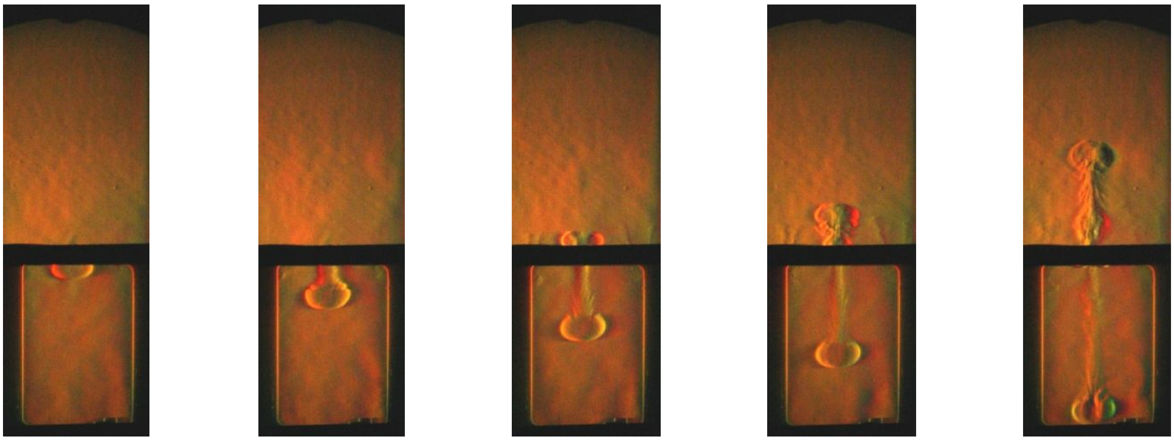
Fig. 2. The scheme of experimental installation on the basis of shadow device IAB–463M: 1 – linear white light source; 2 – adjustable slot–hole diaphragm, 3, 3′ – diagonal mirrors; 4, 5 and 4′, 5′ – elements afocal jacks;, 6′ – collimation mirrors; 7, 7′ – protective plane–parallel glasses collimators; 8 – quadrant Hilbert's phase filter; 9 – digital camera; 10 – generator of vortical rings; 11 – electrodynamic loudspeaker; 12 – computer.

The system of optical diagnostics (Fig. 2) has been executed on the basis of serial shadow device IAB-463M in which the optical filtration block has been modified for the purpose of high–sensitivity colour Hilbert–visualisation of the optical phase density. The quadrant Hilbert filter for a wave length λ=0,63 microns and the slot–hole bichromatic source were used. Registration of the filtered image was made colour CCD by a camera.

Fig. 3 illustrates examples of evolution of the vortical rings induced by the positive (Fig. 3,*a*) and negative (Fig. 3,*b*) pressure jumps on a chamber aperture.



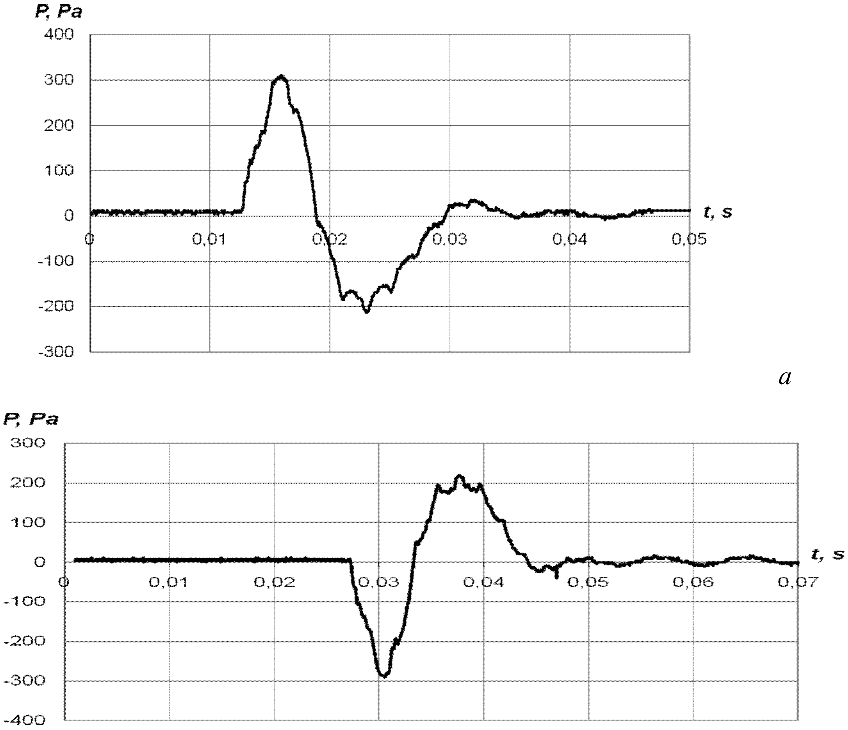
a



b

Fig. 3. Evolution of the vortical rings induced by pressure jump in the gas chamber: a – evolution of pair the vortical rings arising at positive pressure jump; b – evolution of pair the vortical rings arising at negative pressure jump

Impulses of pressure registered on an aperture, are shown on Fig. 4.



a

b

Fig. 4. Pressure evolution on the outlet: a – positive pressure jump; b – negative pressure jump

Fig. 5 illustrates an evolution of the vortical rings induced by the pressure jump whose form is shown on Fig. 6.

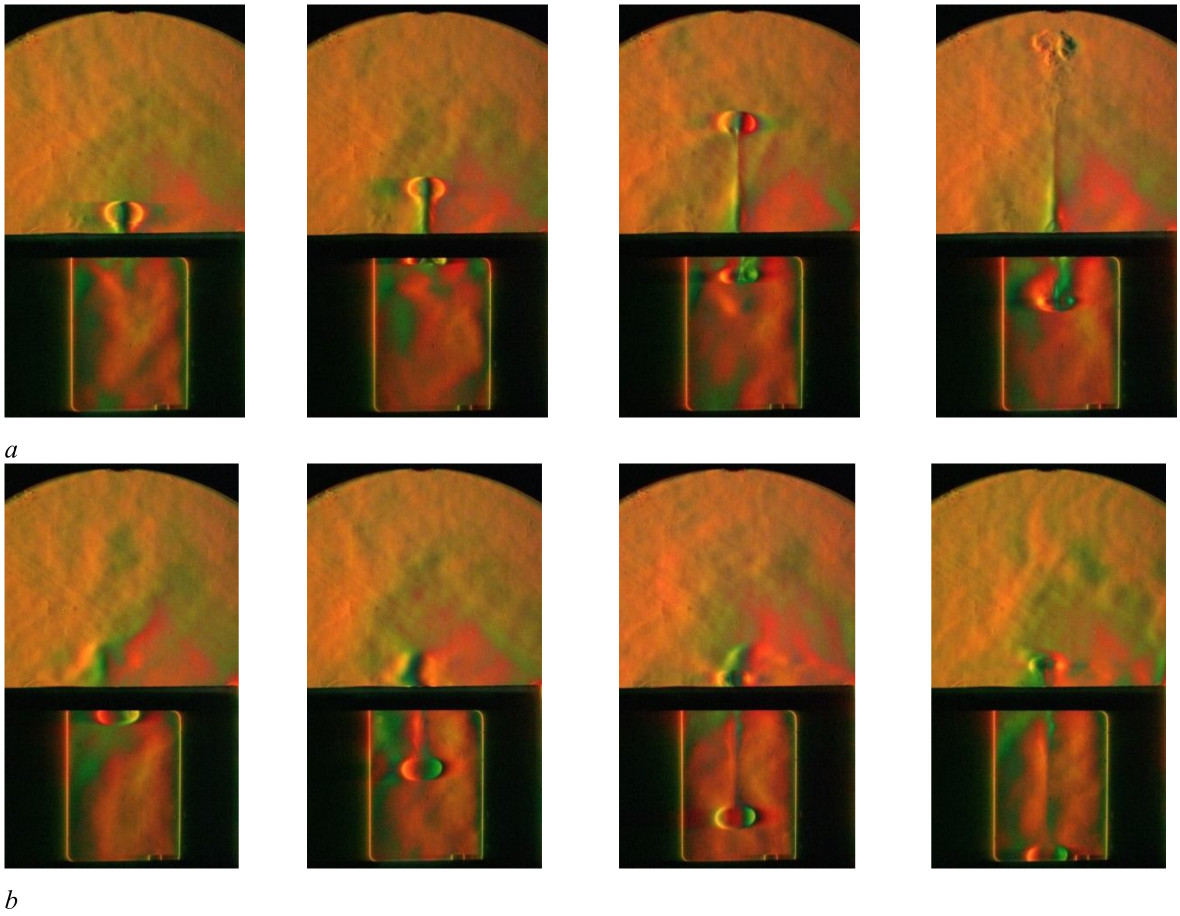


Fig. 5. Evolution of vortical structures: a – at the positive pressure jump (620 Pa); b – at the negative pressure jump on (620 Pa)

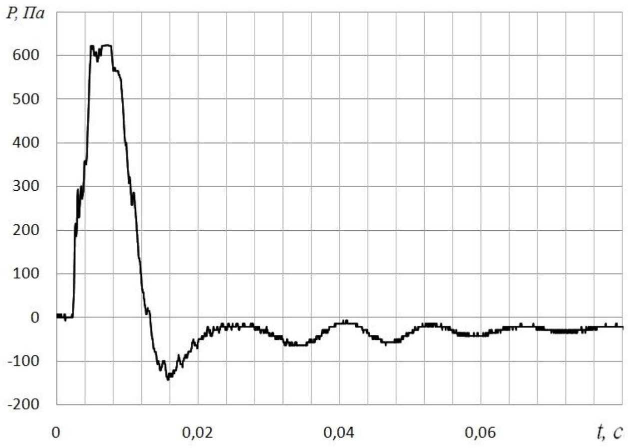
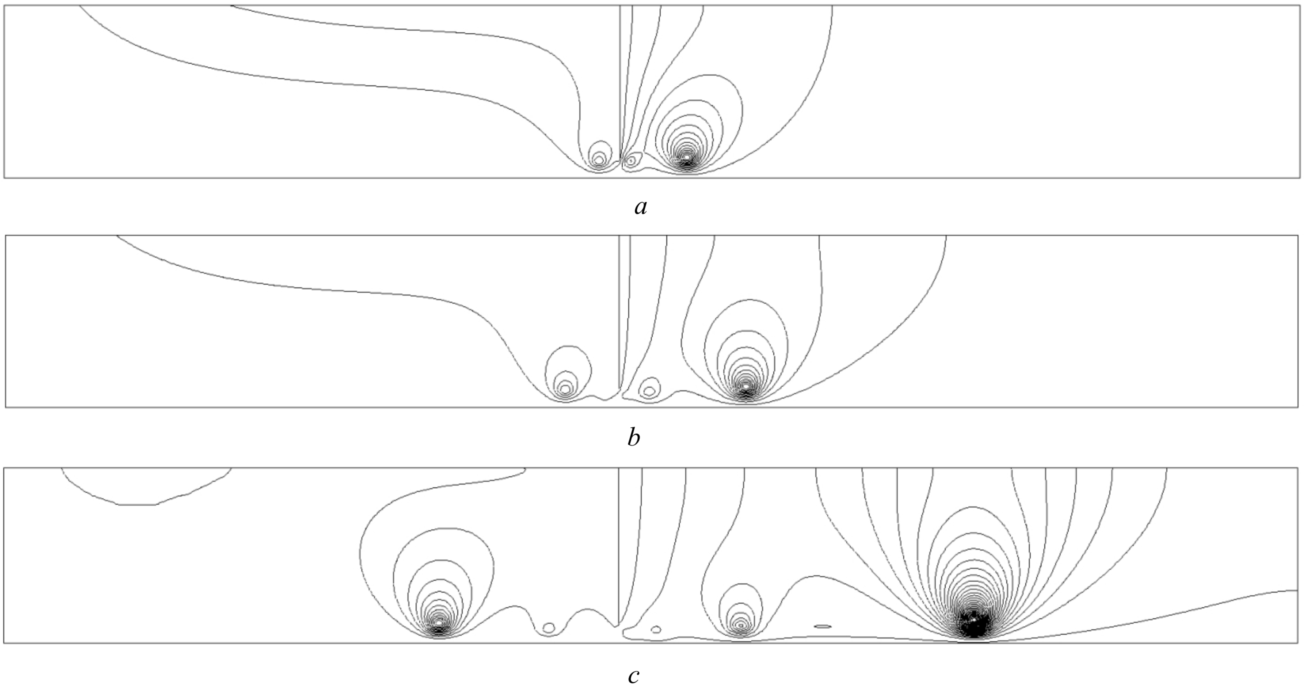


Fig. 6. Dynamics of change of pressure on an the aperture of the gas chamber

Fig. 7 illustrates evolution of the vortical structures induced by the pressure jump which is formed by an elastic diaphragm with binary steady positions at switch from one in another state.

Numerical modelling of vortical structures at sudden depressurization of an aperture (∅20 mm) in the chamber with a gas elevated pressure was carried out on the basis of the non–stationary Nave–Stoks equations for the laminar axisymmetric section. The air pressure in environment P = 101325 the Pa, air temperature 300 T °, initial pressure difference ΔP accepted value 620 Pa. On Fig. 8 calculating lines of a current are presented.



a

b

c

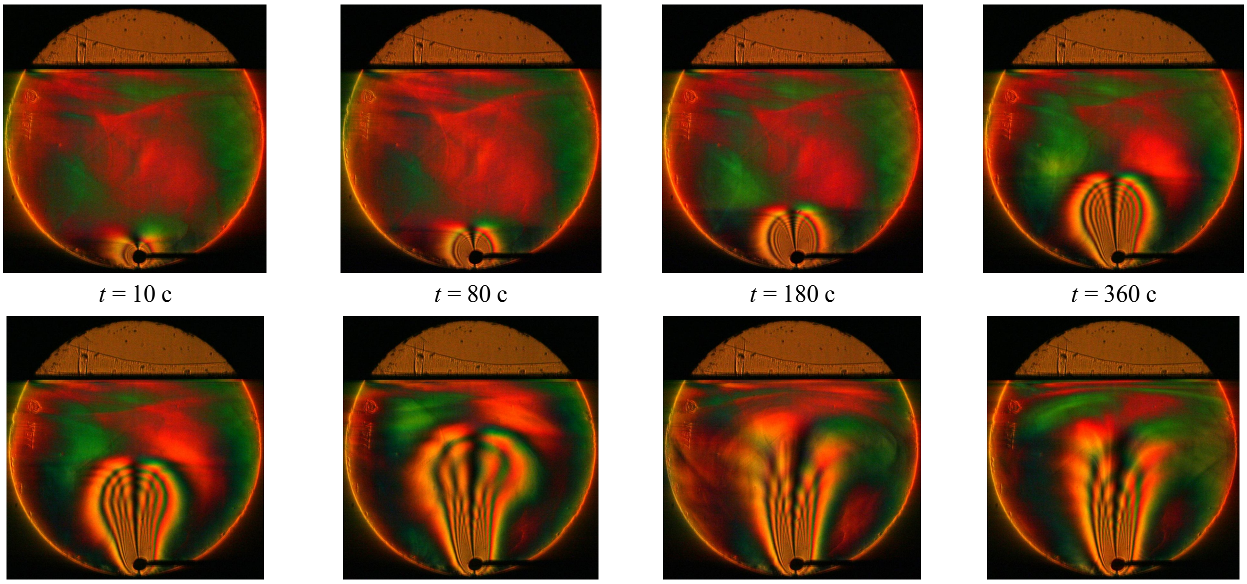
Fig. 8. Evolution current lines for the vortex structures induced by pressure jump (620 Pa) on an aperture in the chamber (on the right an external vortex, at the left–internal one): a – t=0,022 c.; b – t=0,04 c.; c – t=0,12 c. [9].

Results of numerical modelling and experiment have qualitative coincidence. Process of formation of external and internal vortical rings depends on a steepness of the pressure front. At the low steepness the jet mechanism of a birth of whirlwinds prevails, at the high steepness on formation of the vortical rings influence diffpaction phenomens.

3. THERMOGRAVITATIONAL CONVECTION IN THE STRONGLY VISCOUS LIQUID

Researches thermogravitational convection are connected with modelling of geodynamic processes in the mantle on the big depths. The physics of such processes depends from little–studied thermal and rheological properties of the substance in the Earth mantle. An example of physical modelling is research of the formation and evolution of a floating stream over the suddenly included linear source of heat in the strong–viscous polyethylsiloxane liquid PES–5 with Prandtl number P_r=2700 at 20°C [10]. The liquid was located in the rectangular cuvette with transparent walls of optical quality. The internal sizes of the cuvette 300×250×60 мм^2. The heater in a kind constantant wire in (diameter 0,75 mm and length 55 mm) was located in the central section on length on distance of 5 mm from a bottom. The wire heating was carried out by a direct current from the stabilised power supply. It is supposed that in experimental conditions the physical model and real process though differs on many orders on time scales, give the adequate information on the studied phenomenon even in the absence of certain information about properties of substance of the Earth mantle on the big depths.

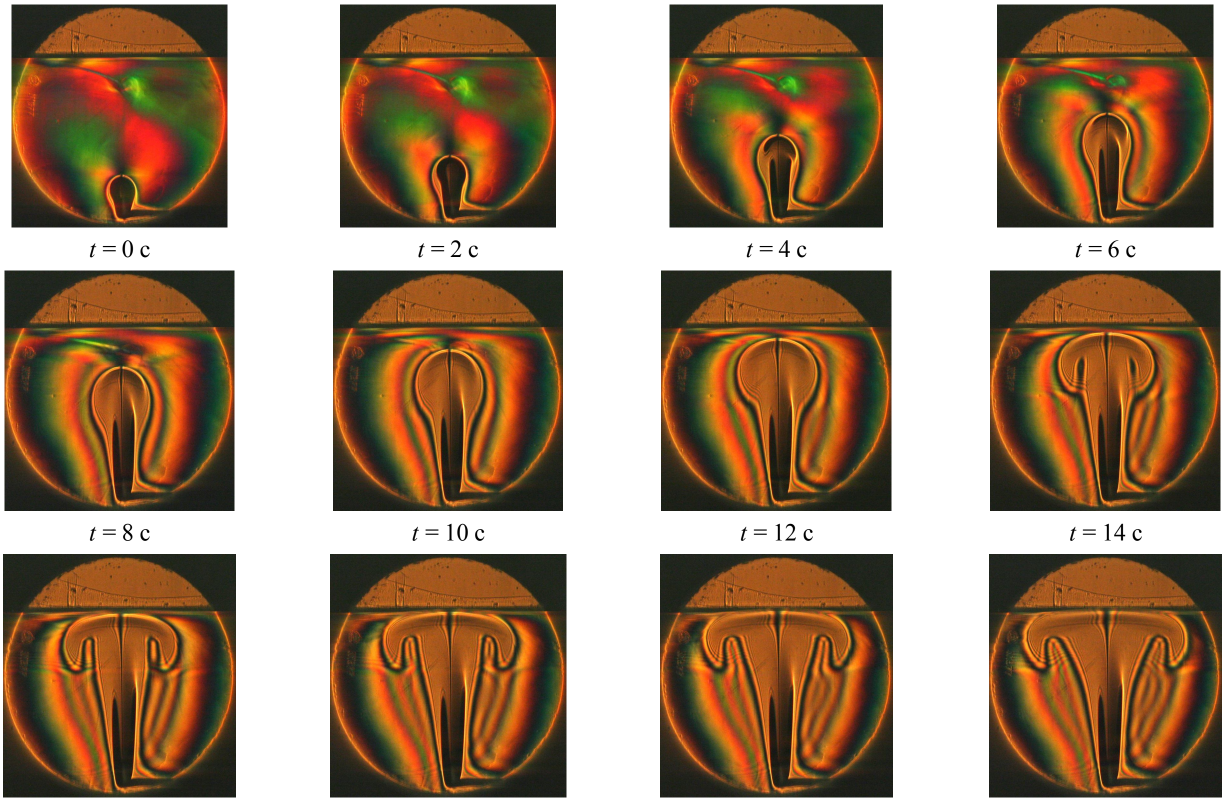
In Fig. 9 and 10 are resulted Hilbert–images of the floating streams, illustrating evolution of thermal indignation at various capacity of the heater. As capacities of the heater were small, it is possible to believe that development of streams occurred in the isothermal environment not stratified on density. Evolution of streams at low power of the heat source depends on height of a layer and properties of a liquid because for formation of a head part of slowly floating stream the certain layer of space over it is required. Decoding Hilbert–image includes a sequence of following procedures: skeletonization of Hilbert–strips in the stream image (program Corel Draw); definition of the initial phase distribution by quantity of Hilbert–strips in the selected section of the stream. Results of processing represent phase distribution in the assigned section of the stream. In Fig. 11 and 12 the decoding example Hilbert–images and temperature profiles on the stream section is shown. The phase profile of the section is transformed to distribution of refractive index which depends on the temperature. From here temperature distribution in the section was defined. The heater switched on in each series of the experiments after damping of the disturbances and temperature alignment.



$t = 10$ c $t = 80$ c $t = 180$ c $t = 360$ c

$t = 480$ c $t = 720$ c $t = 1200$ c $t = 1800$ c

Fig. 9. Evolution of thermal disturbance in poliethylsiloxane liquid PES–5 (power 0,41 W/m)

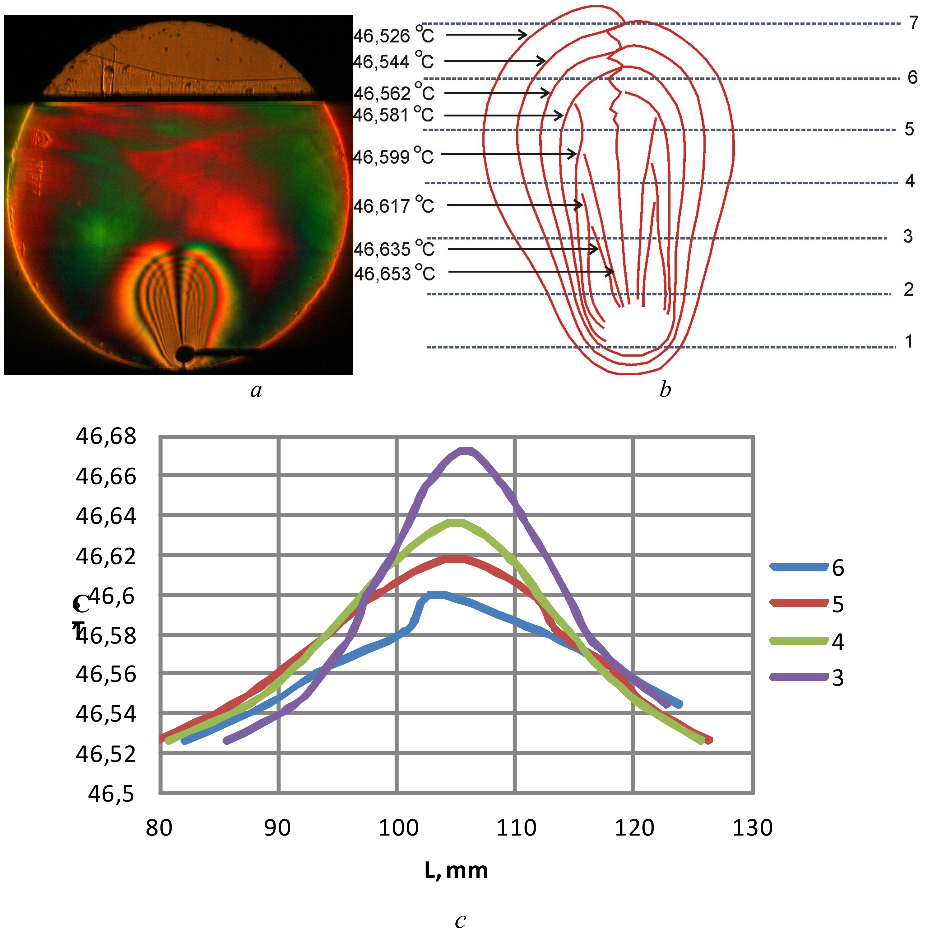


$t = 0$ c $t = 2$ c $t = 4$ c $t = 6$ c

$t = 8$ c $t = 10$ c $t = 12$ c $t = 14$ c

$t = 16$ c $t = 18$c $t = 20$ c $t = 22$ c

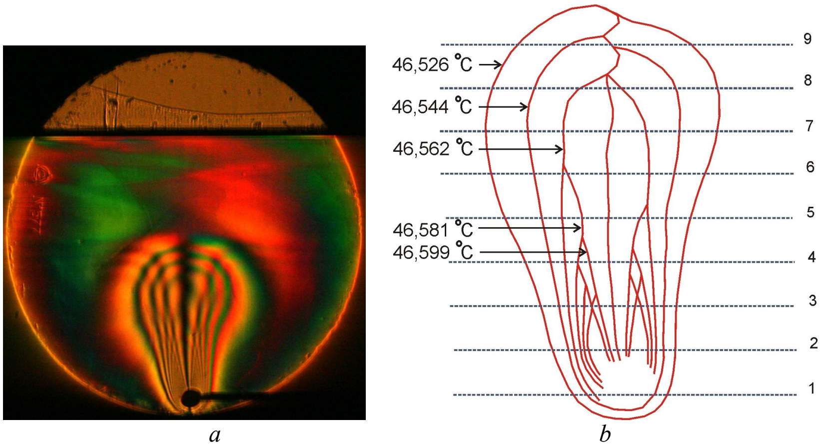
Fig. 10. Evolution of thermal disturbance in poliethylsiloxane liquid PES–5 (power 28 W/m)



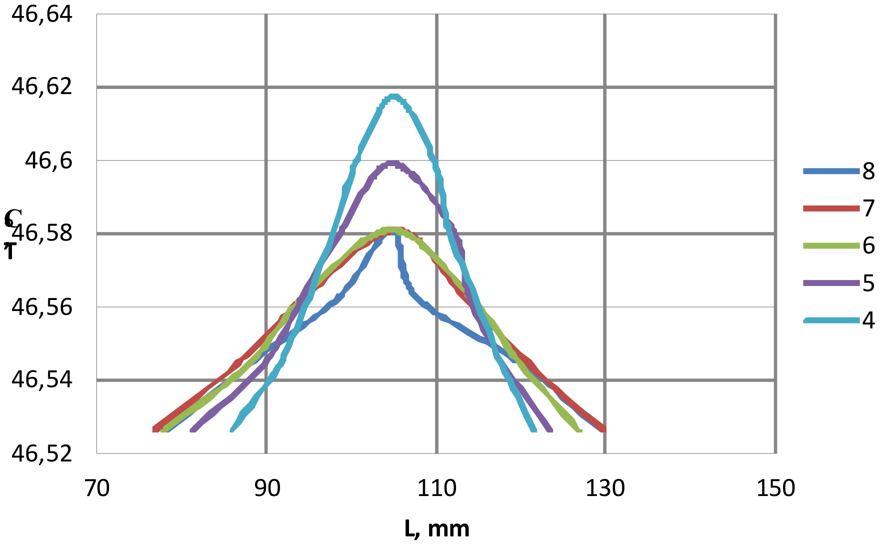
a *b*

c

Fig. 11. Temperature distribution in the floating stream (poliethylsiloxane liquid PES–5, a power 0,41 W/m, t = 6 min): a – Hilbert–image; b – Hilbert–strips skeletonization; c – a temperature profile in stream sections



a *b*



c

Fig. 12. Temperature distribution in the floating stream (poliethylsiloxane liquid PES–5, a power 0,41 W/m, t = 8 min): a – Hilbert–image; b – Hilbert–strips skeletonization; c – a temperature profile in stream sections

Results of research of floating streams can be used for improving rheological model of strong–viscous liquids at extremely low velocities.

CONCLUSION

Bichromatic Hilbert–optics methods allow to carry out high sensitivity colour visualisation of fields of the phase optical density in gases and liquids at the research of currents structures. With application of the optical Hilbert–visualization the evolution of complemehtary vortical structures induced by the pressure jump on an aperture and extending inside and outside of the gas chamber is investigated. Within the limits of physical modelling rheological processes in deep layers of the Earth mantle Hilbert–visualisation methods are used for research of floating streams evolution over suddenly included linear source of the heat in the strong–viscous liquid. Possibility reception of the temperature distribution in stream sections with using structure of Hilbert image is shown. The received results illustrate an efficiency of Hilbert–optics applications in experimental hydro– and gas dynamics.

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