QUANTITATIVE MEASUREMENTS OF THE DENSITY GRADIENT ON THE FLAT SHOCK WAVE BY MEANS OF BACKGROUND ORIENTED SCHLIEREN

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Introduction.

The paper presents the results of experimental quantitative analysis of shock waves in the shock tube by means of background oriented schlieren (BOS) [1] and the particle image velocimetry (PIV) methods.

The results of previous experiments, as well as works by other scientific groups, show that quantitative capturing of the density jump on a shock wave with the BOS in its conventional scheme poses a difficult problem [2][3]. It was shown that the problem arises from the limitations on the density gradient being captured by refraction and the BOS processing. In order to solve these problems, a modification of the optical scheme and image processing sequence is proposed and tested in this work. Three various approaches to image processing are compared. Another effective and widely applied optical technique providing quantitative data is the PIV method [4]. Velocity measurements performed by this technique are effectively independent in each point of the flow image, and the effects of the shock wave front do not affect other areas of the field, as it happens with BOS measurements. In this paper results of PIV imaging of an expanding shock wave are presented. Shock wave thickness recorded with the methods is analyzed.

Background oriented schlieren.

The essence of the BOS method is comparison of two images of the same background, taken with (working image) and without (reference image) the investigated transparent object between the camera and the background .

The variation of the medium refraction index in the image plane inside the phase object leads to the deflection of the light ray and to distortion of the working image relative to the reference image (Fig. 1). By analyzing the shift of the background elements it is possible to obtain quantitative information on the refractive index n of the investigated medium, averaged along the optical path.

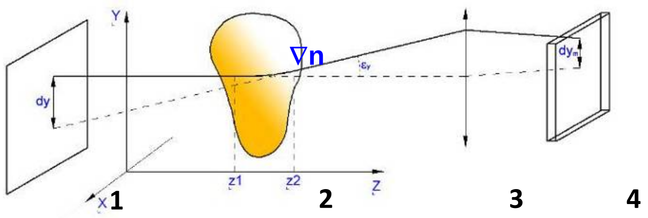

Fig.1. The conceptual scheme of the BOS method: 1) background, 2) object under investigation, 3) lens, 4) CCD/CMOS sensor.

The relation between density and refraction index of gas is expressed by Gladstone-Dale equation:

$$\frac{n-1}{\rho} = G\,,$$

where G is the Gladstone-Dale constant, specific to the given medium. If we consider the displacement being detected on a digital image, then the final expression for the density field can be written in the form of Poisson equation:

$$\frac{\partial^2 \rho}{\partial a^2} + \frac{\partial^2 \rho}{\partial b^2} = -\frac{2R_b R_o}{Gh(2L_b + h)}\left(\frac{\partial p_x}{\partial a} + \frac{\partial p_y}{\partial b}\right), \quad (1)$$

where R_b and R_o correspond to one pixel size in background plane and the phase object plane, $a = x / R_o$ and $b = y / R_o$ are coordinates in image measured in pixels, p_x and p_y are displacement components, also measured in pixels. In general, the quantitative data on the density of the flow under investigation is obtained by solving (1). Further, calculation of temperature or other parameters is possible using the state equations of the medium [2][5]

Shock wave in shock tube channel.

The gas density increase $\Delta\rho$ on the shock front is an essential parameter of the flow. The parameters on the shock are governed by the Rankine-Hugoniot equations. The equation for the gas density is as follows:

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2+2}$$

If we consider a shock wave with M=2 propagating through ambient air at a low pressure of 75 Torr, the density of the gas passing through the shock is multiplied ≈2.6 times. The refractive index of the gas increases respectively, from 2.96×10^{-5} to 7.8×10^{-5}. As the thickness of the shockwave front is of 10^{-7} cm magnitude, the value of the gradient of the refractive index, calculated perpendicular to the shock surface, reaches 10^2 cm^{-1}. This value exceeds upper resolution threshold of the most BOS imaging systems and algorithms; it requires refitting of the optical scheme and application of sophisticated processing algorithms for capturing. The upper threshold is imposed by the cross-correlation algorithm commonly used for the processing of the experimental image pairs. The maximal value of the pixel displacement that can be captured by the cross-correlation scheme is usually less than the size of the request area used in the processing. However, the area of the gradient localization poses even a greater problem, as it is virtually impossible to achieve digital resolution of such small-scale features. The abovementioned limitations lead to significant errors while attempting to measure the density jump on the shock front using the BOS method [6]. The density field calculated from the displacement differs from the results of theoretical assessments and direct measurements.

To overcome the noted difficulties, a change in the BOS scheme is proposed. The optical axis of the BOS system (basically the optical axis of the camera) is placed at an angle γ to the shock surface, in the same horizontal plane. As the shock surface in the shock tube channel is a rectangle equal to the channel cross-section, the angular size of the shock on the image increases, and the gradient appears to be localized in a wider area of the image. Also the refraction of light passing through the phase object is simplified significantly in case of a flat shock wave.

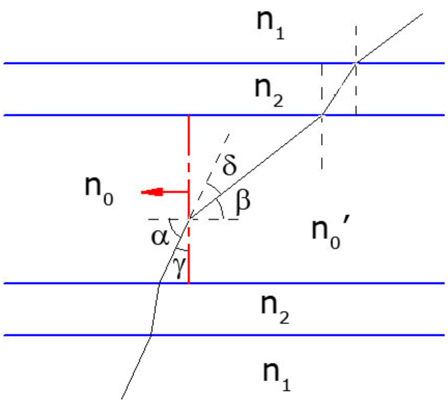

Fig. 1. Schematics of light refraction on a flat shock wave (red) propagating in the channel

If we consider the shock front as the media interface, then Snell's law can be applied to calculate the refraction of the light rays. For the conditions of the experiment $\frac{\Delta n}{n} << 1$, the angles δ, γ are small, and the refraction angle can be expressed as:

$$\delta = \frac{\Delta n}{\sin \gamma}$$

Here the refraction on the glass walls is taken into consideration, but as for air $n \approx 1$, this does not affect the final expression for the registered angle of deflection.

As the parameters of the flow after the shock can be considered homogeneous, it is possible to determine the variation of the refractive index (and the density) on the shock, using the displacement values only, with no need to reconstruct the refractive index field by solving the Poisson equation. Indeed, if we consider the relation between the real width of the shock h (here equal to the width of the channel) and its visible width d, then

$$\delta = \frac{\Delta n \cdot h}{d}$$

The variation of the refractive index can be expressed with the visible background displacement p_x:

$$\Delta n = \frac{d \cdot R_b p_x}{h\left(L_b + \frac{h}{2}\right)},$$

and the final expression for the density jump on the front of the shock wave is:

$$\frac{\rho_1}{\rho_0} = \frac{d \cdot R_b p_x}{Gh\left(L_b + \frac{h}{2}\right)} + 1 \qquad (2)$$

In this case, the density variation can be determined using only the values of the background displacement, the gas constants and the geometry of the experimental set-up.

Experimental set-up

In this study the shock waves were generated in the shock tube. The driven section is 250 cm long, with a rectangular cross-section of 24×48 mm. The driver gas used in the experiments is helium, with air as the driven gas. The experiments were conducted with a low pressure of 75-100 Torr. The Mach number of the shock wave varied from 1.8 to 2.2 through the series of experiments. The test section has the side walls with 16 mm thick quartz glass, retaining the inner geometry of the driven section. The illumination of the background was provided by a Nd:YAG laser with the pulse duration of 20 ns (534 nm). The image of the illuminated background was captured by Canon 500D digital SLR camera with a 75 mm lens attached. The observation angle γ varied from 0° to 20°, to test the proposed imaging method.

On digital processing of the experimental images, the displacement field was determined by a multi-scan variant of cross-correlation method. Rectangular non-overlapping request areas were used, with their sizes being halved on each successive pass of the algorithm. For noise reduction a normalized median filter was applied to the displacement field, and on the last pass the displacement was approximated using the Gaussian peak function to obtain subpixel precision (ссылку либо выкинуть?).

Experimental results

Fig. 2 presents the results of BOS imaging of the M=2,21 flat shock wave, propagating in the tube channel through the ambient air at $p = 75$ torr from right to left. The shock has been captured at an angle $\gamma \approx 20°$. The displacement field shows the gradient being extended to a wide area. The width of this area is bigger than expected from the geometrical calculations. The deviation most likely arises from the light diffraction on the edges of the shock, and boundary effects.

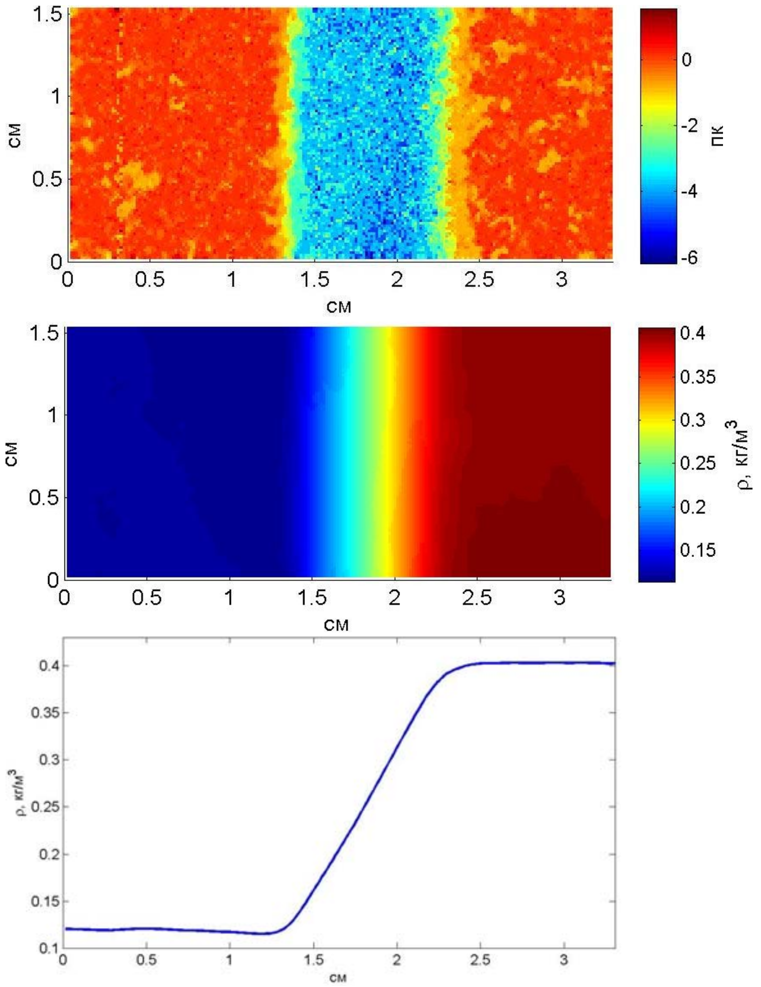

Fig. 2. Horizontal displacement field (top), calculated density field (middle), horizontal density profile (bottom) of the flat shock wave acquired by BOS method.

The density field shows a smooth increase in the gradient area, but the values of the density leap differ from the results of the Rankine-Hugoniot equation, the BOS results being approximately 30% bigger. Of course, one cannot expect reliable results of density calculation from (1), as this approach requires the density gradient to be constant along the optics ray, which is not true for this case.

Three different methods have been used in this work to calculate the $d \cdot p_x$ value in (2):

1) the displacement is determined by visual comparison of the BOS image pairs. The p_x value is then calculated as the mean value of the central part of the gradient area, where the effect can be considered uniform. The visible shock width d is calculated from the geometry of the experiment;

2) similar to the first method, but the displacement is determined by the computer cross-correlation algorithm;

3) the displacement is determined by digital processing, but then the integral $\int p_x dx$ through the whole gradient area is calculated instead of the $d \cdot p_x$ value.

Results of the density calculation

Fig. 3 represents the data on the density ratio $\Delta = \rho_1 / \rho_0$ on the shock front. The theoretical values were derived from the Rankine-Hugoniot equation using the known Mach number for each experiment, and each of the experimental images has been processed by the three methods described. The errors for the theoretical Δ estimates and the angle calculation are shown for a single black point in the left part of the graph.

The error varies with different versions of the processing sequence, and is also angle-dependent. The versions 1 and 2 results depend heavily on the precision of the γ value, as the observation angle is being used for the calculation of the visible shock width d . This error increases at smaller observation angles.

The error of the method variant 1) is high because of the uncertainties of visual detection of the image displacement. However, digital algorithms show less stability at lower angles ($\gamma \leq 2°$), where image distortion due to the fringe effects becomes significant. Bigger observation angles lead to the decrease of the absolute displacement being registered, and the increase of the relative errors. All three methods for calculation of the density rise give results exceeding the outcomes of the Rankine-Hugoniot equations in average.

Former studies on the effectiveness of the crosscorrelation algorithm show that the best precision is obtained for intermediate values of the displacement being detected. For the conditions of the experiment used in this work, the optimal parameters correspond to the observation angles between 8° and 11°. Indeed, the best correlation between the theoretical calculations and the BOS data is obtained for $\gamma = 8° - 10°$.

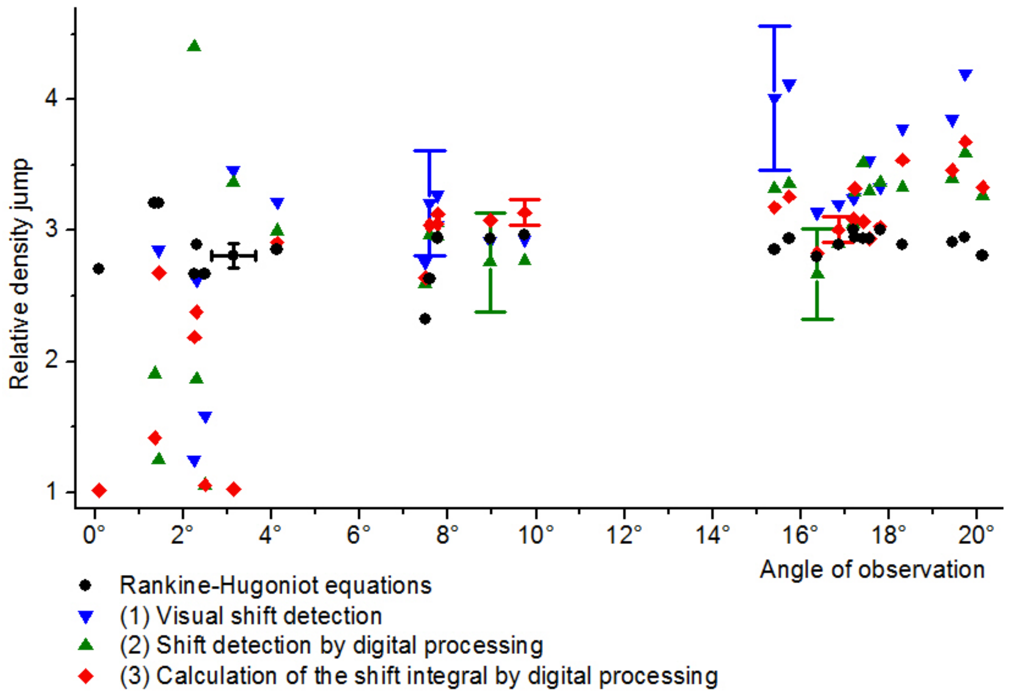

Fig. 3. Calculation of the relative density jump on the front of a flat shock wave

PIV measurements

Another optical method used was particle image velocimetry (PIV) method. As the BOS method, it also employs digital processing to obtain the quantitative data, and the cross-correlation processing algorithm is also used in PIV applications. The method has been already used for studies of shock processes [7], and the complex flow generated by the diffraction of the shock wave on an open end of the shock tube [8][9]. In this work PIV was applied to the studies of an expanding shock wave front. The shock tube was reconstructed for these experiments. The test section was not used, and the low pressure chamber was opened to the atmosphere, with the PIV being set up at the open end. A double-frame LaVision PIV imaging system was used with the Litron dual laser as a light source. The laser sheet was placed parallel to the flow axis, to visualize the axial cross-section of the flow. The time between the 2 frames was set to 1 μs. Aerosol of dioctyl sebacate was used as seeding particles.

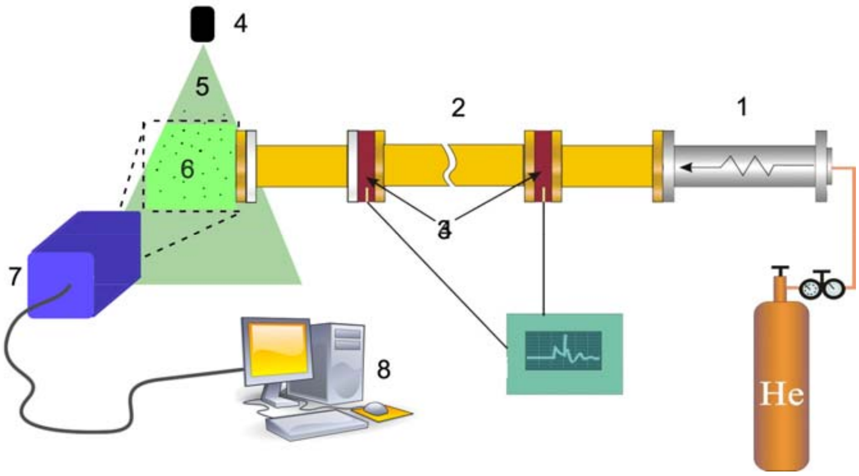

Fig. 4. Experimental set-up for the PIV measurements. 1 – high pressure chamber, 2 – low pressure chamber, 3 – pressure gauges in the tube channel, 4 – laser head, 5 – laser sheet, 6 – frame area, 7 - camera, 8 – controller PC.

Fig. **5** presents the velocity field for the flow originating after a shock wave has left the channel. The vortex ring and the axial jet were clearly visualized. The shock wave front is seen as a line separating moving gas from quiescent air. The original image, shown in the background in grayscale, shows also the position of the contact sheet enveloping the left part of the flow.

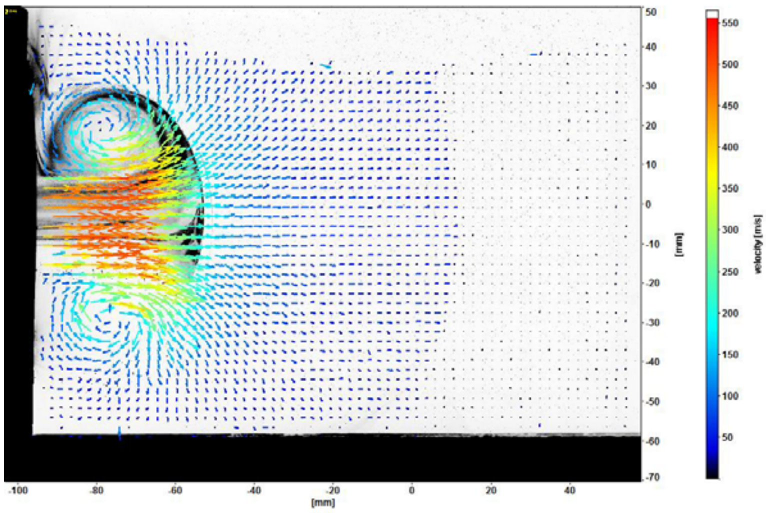

Fig. 5. Velocity vector field calculated by PIV

Fig. **6** presents the PIV velocity profile for an earlier stage of the process. The velocity jump corresponding to the shock front can be measured, and the series of experiments show results well corresponding to theoretical expectations. The width of the slope on the velocity profile decreases with the shock expansion from ≈4,2 mm to ≈2 mm.

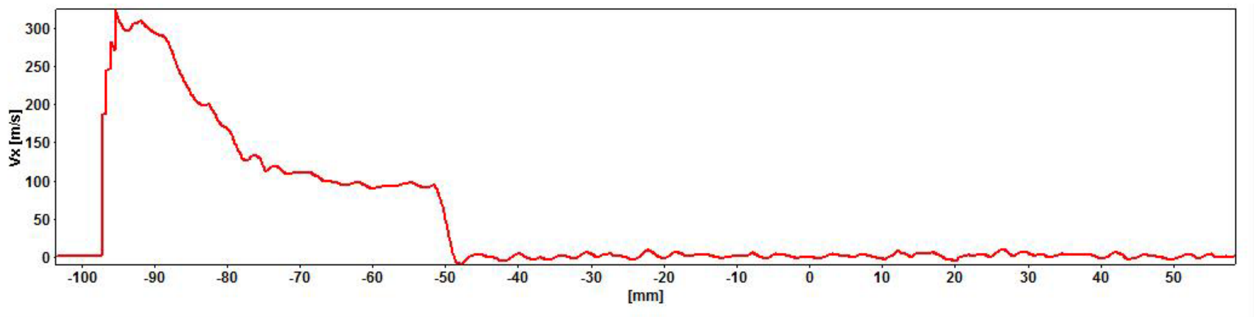

Fig. 6. Axial velocity profile

Conclusion

Two different optical methods involving complicated image processing have been used for quantitative studies of the parameters on the shock wave front. The density measurements performed by the BOS method required modifying the classical scheme and processing procedure to allow resolving of the flat shock wave front. The best correlation between the theoretical calculations and the BOS data is obtained for $\gamma = 8° - 10°$. The visible width of the shock front at these angles of observation equals 8-9 mm. The technique still needs adaptation to improve accuracy of the measurements.

The measurements performed by the PIV system have yielded reliable quantitative data on the velocity field and the velocity jump on the front of a shock wave expanding in the atmosphere after exiting the rectangular nozzle. The Mach number of the shock wave in the channel was M=1,4-2. The front of the expanding shock is shown to be recorded by PIV method as being 2-4 mm wide. Further numerical simulations and analysis can provide reference data for comparison.

Acknowledgements

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