



## VORTEX SHEDDING AND VORTEX FORMATION FROM A PAIR OF IN-LINE FORCED OSCILLATING TANDEM ARRANGED CIRCULAR CYLINDERS IN A UNIFORM FLOW

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### KEYWORDS:

**Main subjects:** experimental fluid mechanics, flow visualization

**Fluid:** wakes, separation

**Visualization method(s):** streak line tracer method, laser light sheet

**Other keywords:** vortex, lock-in, in-line oscillation, tandem arranged cylinders

**ABSTRACT:** In this study, the flow features of vortex shedding from a pair of tandem arranged circular cylinders oscillating along the direction of the flow were observed by visualizing water flow experiment at the ranges of the frequency ratio  $f/f_k=0\sim 7$ , the amplitude ratio  $2a/d=0.25, 0.5, 0.75$  and  $1.0$ , the distance ratio  $L/d=1.5, 2.5$  and  $5.5$  and Reynolds number  $Re=640$ . The variations of mean vortex shedding frequency were investigated. Although the cylinder oscillation frequency  $f$  is lower than the natural Karman vortex's frequency  $f_k$ , the lock-in phenomenon can be seen even in tandem arrangement case. The lock-in ranges have been shown in each distance ratio  $L/d$ . It is found that the lock-in ranges become wide when comparing with the single cylinder case. The typical flow patterns of the lock-in or un-lock-in states were shown every distance ratio  $L/d$ . In distance ratios  $L/d=2.5$  and  $5.5$ , the twin vortex street from the 2nd cylinder (rear set cylinder) was not seen.

**INTRODUCTION.** The flow induced vibration problem is very important problem for engineering field so that there are many reports and useful knowledge has been accumulated by many researchers. The "lock-in" is the phenomenon which the vortex shedding frequency from a cylinder synchronizes with the frequency of cylinder vibration. And it is one of most interesting phenomenon in the field of flow induced vibration. Okajima [1, 2] reported the review and the view about the flow-induced vibration including the lock-in phenomenon. Yokoi and Hirao [3] investigated the effect of cylinder oscillation on vortex shedding and they classified flow pattern by flow visualization into five kinds by the configuration of vortex shedding and direction of vortex shedding. Although there are many studies about the lock-in region and the flow patterns of lock-in border in the case of single cylinder, there are few studies which discuss about the case of plural cylinders. Moreover, there are few reports which example to have examined to use a lock-in phenomenon effectively industrially. In order to examine the industrial use of the lock-in phenomenon, the lock-in range and the state of the flow at the time of lock-in must be more widely known in detail. Therefore, the systematic and scrupulous investigation by which the range of oscillation frequency and amplitude expanded becomes necessary, and the database which accumulated the result is required. Yokoi and Hirao [4, 5] investigated the feature of vortex shedding and the vortex formation from a pair of in-line forced oscillating parallel arranged circular cylinders of same or unequal diameter. They show the variation of mean vortex shedding frequency, the "lock-in" pattern distribution in the lock-in region and representative flow patterns. In this study, the flow patterns of vortex shedding from a pair of tandem arranged circular cylinders oscillating along the direction of flow were investigated by visualizing water flow experiment.

**EXPERIMENTAL APPARATUS AND METHOD. Experimental Apparatus;** The experimental apparatus consists of a closed circuit water channel, a cylinder oscillator, a set of visual apparatus and set of data record equipment. The closed circuit water channel is a vertical circulation type of 5.8m in length, 1.2m in width, and 2.5m in height, and the volume of water is 4 tons. The water channel is consisted of water flow generation equipment (2 axial flow type pumps), rectification device, test section and 4 corner parts with guide vanes. The test section is 2m in length, 0.8m in width, 0.4m in depth, and the flume structure with the surface of the water. In the test section, the window made of the glass of 1.5m in length and 0.4m in width has been installed in the both sides wall and the bottom for the observation.

The velocity distribution in the test section was  $\pm 1.5\%$  against main flow velocity  $0.4\text{m/s}$  from depth  $50\text{--}350\text{mm}$  and the test section center in the direction of width within the range of  $\pm 300\text{mm}$ . There is a set of rail orbit on the flume flange and a carriage is set up. The oscillator which installed CCD (charge-coupled device) camera and test cylinder, the visualization apparatus are loaded into the carriage. The cylinder oscillator is using a Scotch-yoke mechanism and the oscillator consists of a small-sized variable DC motor with a controller, a turn disk and a connecting rod. The amplitude of oscillation is set by changing the rotation radius of connecting edge on the turn disk, while the frequency of oscillation is controlled by changing the revolution speed of the turn disk. Two kinds of circular cylinders were used. One of which was object for a single cylinder test, and the other was object for two cylinders test. The circular cylinder for the single cylinder test is made from acrylic resin with  $25\text{mm}$  of out side diameter,  $19\text{mm}$  caliber, and  $600\text{mm}$  length. Four tracer oozing ports are provided on the cylinder surface. Tracer oozing ports are in plus-minus  $60$  degree from the upstream side stagnation point and the down stream side stagnation point, respectively. The circular cylinders for the two cylinders test are made from hollow aluminium with  $16\text{mm}$  of outside diameter,  $14\text{mm}$  calibres, and  $600\text{mm}$  length. The cylinders of them without end plates are mounted vertically in a free-surface water channel, where its bottom end is free and its top end is clamped to an oscillator. Thus the circular cylinder is given a longitudinal sinusoidal oscillation. The arrangement of circular cylinders is shown in Fig. 1. For convenience, front side and rear side cylinders are called the “1st cylinder” and “2nd cylinder”, respectively.

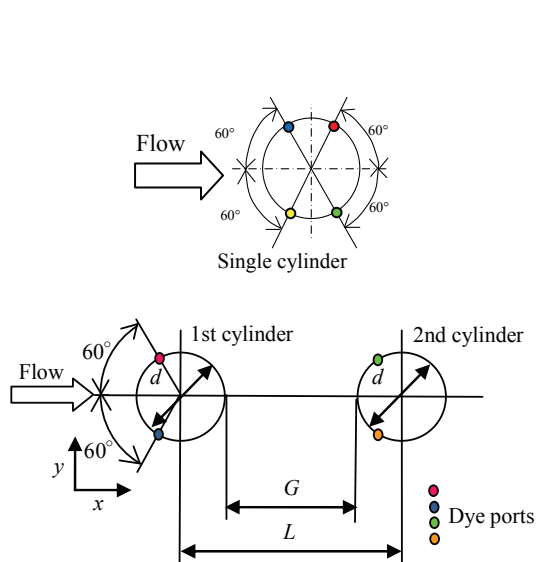


Fig. 1 Coordinate system and definition of symbols

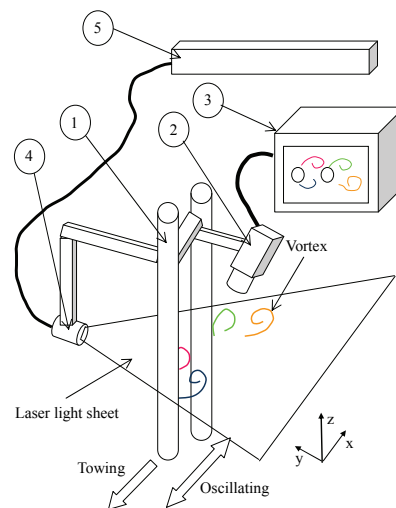


Fig. 2 Schematic diagram of the layout of flow visual apparatus;  
 1. Circular cylinder, 2. CCD video camera,  
 3. Monitor with recorder, 4. Cylindrical lens,  
 5. Laser unit

**Flow Visualization Method;** A schematic diagram of a layout of flow visual apparatus is shown in Fig. 2. The horizontal section of the flow at a depth of  $140\text{mm}$  below the water surface which was enough for observation was visualized by the laser light sheet technique which was based on the dye injection method. The visualized flow patterns were monitored by a CCD video camera which set above the surface of the water. In this experiment, an argon gas laser beam ( $4\text{W}$  maximum power) was conducted to a cylindrical lens, which spread it into a two-dimensional laser light sheet. In the plane of the laser light sheet, the flow was visualized by means of tracer ink (Rhodamine B and poster paints (turquoise, fluorescent pink, fluorescent green and fluorescent orange) with same specific gravity of water), which oozed out from two dye ports at plus-minus  $60$  degree from the front stagnation point of the circular cylinder. The visualized flow patterns were monitored by a CCD video camera and recorded on video tapes. In the single cylinder test, it also performed making a tracer ooze from four oozing ports, respectively.

**Experimental Parameters;** The main experimental parameters given by the distance ratio  $L/d$  (the ratio of cylinder distance  $L$  to the outside diameter of cylinder  $d$ ), the oscillation frequency ratio  $ff_K$  (the ratio of circular cylinder oscillation frequency  $f$  to natural Karman vortex's frequency  $f_K$ ), the amplitude ratio  $2a/d$  (the ratio of half amplitude of cylinder motion  $a$  to the outside diameter of cylinder  $d$ ) and the main flow velocity in the test section  $U$ . The distance ratio  $L/d$  was changed three stage ( $L/d=1.5, 2.5$  and  $5.5$ ). The oscillation frequency ratio  $ff_K$  was varied from  $0.0$  to  $7$

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(28 steps), and the amplitude ratio  $2a/d$  was set 0.25, 0.5, 0.75 and 1.0. The main flow velocity  $U$  was 0.041m/s which correspond to Reynolds numbers about 640 and 1000 ( $Re=Ud/\nu$ , where  $\nu$  is the kinematic viscosity of water).

**Experimental Procedure;** At first step, the flow velocity of the test section is set up by inputting operation frequency into the operator control panel of the closed circuit water channel. The second step, the tracer ink was oozed into the flow from two dye ports on a stationary circular cylinder and the visualized aspects of separated wake flow was recorded on video tapes. The natural Karman vortex shedding frequency  $f_K$  was measured by counting the visualized vortices shed from a stationary circular cylinder for a certain period of time. (In this experiment, the values of Karman vortex frequency  $f_K$  were 0.45Hz and 0.29Hz, respectively.) The third step of experiment is the measurement of the vortex shedding frequency  $f_{VK}$  from each in-line oscillating circular cylinder. The cylinder distance  $L$ , the half-amplitude of oscillation  $a$  and the cylinder oscillation frequency  $f$  were set to be desired values of the distance ratio  $L/d$ , the amplitude ratio  $2a/d$  and the oscillation frequency ratio  $f/f_K$ . The tracer ink was oozed and the visualized flow feature from the oscillating cylinders was monitored and recorded on video tapes. The vortex shedding frequency  $f_{VK}$  was obtained from the number and the measurement time of past vortex at the observation point. The cylinder oscillation frequency  $f$  was calculated by measuring an oscillation cycle.

**EXPERIMENTAL RESULTS AND DISCUSSION. Flow Patterns of Lock-in States of Oscillating Single Cylinder;** The phenomenon in which the vortex shedding frequency  $f_{VK}$  from a circular cylinder synchronizes with cylinder oscillation frequency  $f$  is called lock-in phenomenon. Figure 3 shows the influence of oscillation on a vortex shedding as an example. An abscissa is an oscillation frequency ratio  $f/f_K$  and an ordinate is a vortex shedding frequency ratio  $f_{VK}/f_K$ . Two solid lines in the figure mean the lock-in. One is 1 time of a vortex shedding at oscillating 2 period, and another is 1 time of a vortex shedding at oscillating 1 period. The former is called '1/2 lock-in', and the latter is called '1/1 lock-in'. It means that the dashed line in a figure does not have the influence of oscillation seemingly. The typical flow patterns of lock-in state are shown in Fig. 4. Figure 4 (a) is in '1 / 2 lock-in' state, vortices are shed alternately and the large-scale vortex of mushroom section shape is formed in the wake. Figure 4 (b) is in '1 / 1 lock-in' state, vortices are shed simultaneously and the twin vortex street is formed.

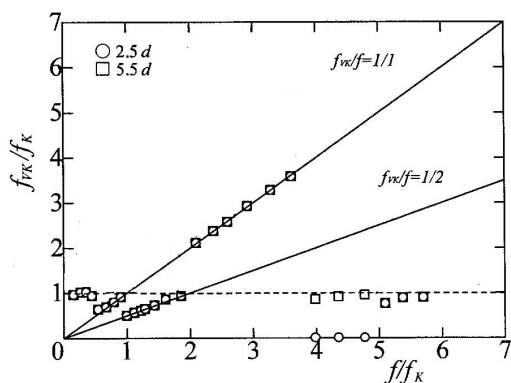


Fig. 3 Variation of mean vortex shedding frequency ratio in the case of single cylinder;  $2a/d=0.44$

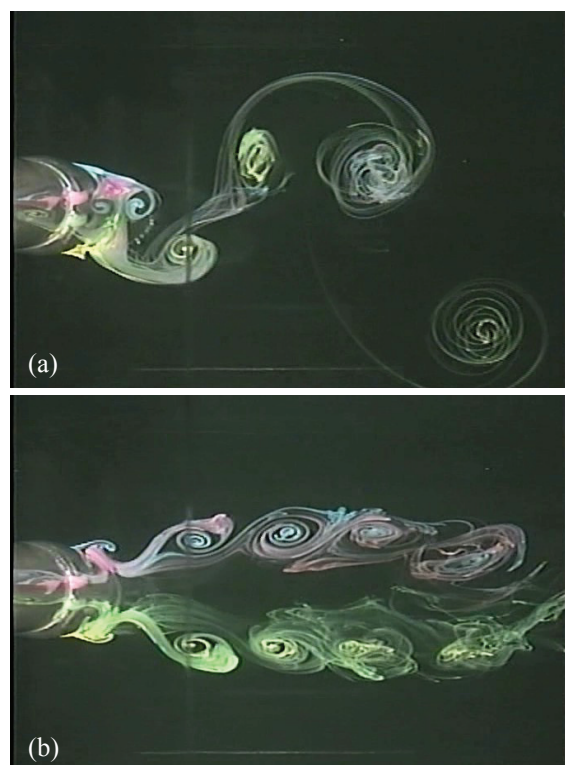


Fig. 4 Representative flow patterns in the lock-in state; (a)  $f_{VK}/f=1/2$ , (b)  $f_{VK}/f=1/1$

**Flow Feature in the Oscillating Tandem Cylinders;** The sketches of the vortex shedding situation from a circular cylinder are shown in Fig. 5. Here, the vortex shedding of an alternate situation was called 'A' (Alternate), and the vortex shedding of a simultaneous situation was called 'S' (Simultaneous). The case where the direction of a vortex shedding changed like a pendulum was named 'P' (Pendulum), the case where a direction was alternately changed at the time of a vortex shedding was called 'AP', and the case where a direction was simultaneously changed at the time of a vortex shedding was called 'SP'. And the case where the separating shear layer and vortex which are discharged from the 2nd circular cylinder were contained in the wake of the 1st circular cylinder was called 'IW' (In Wake).

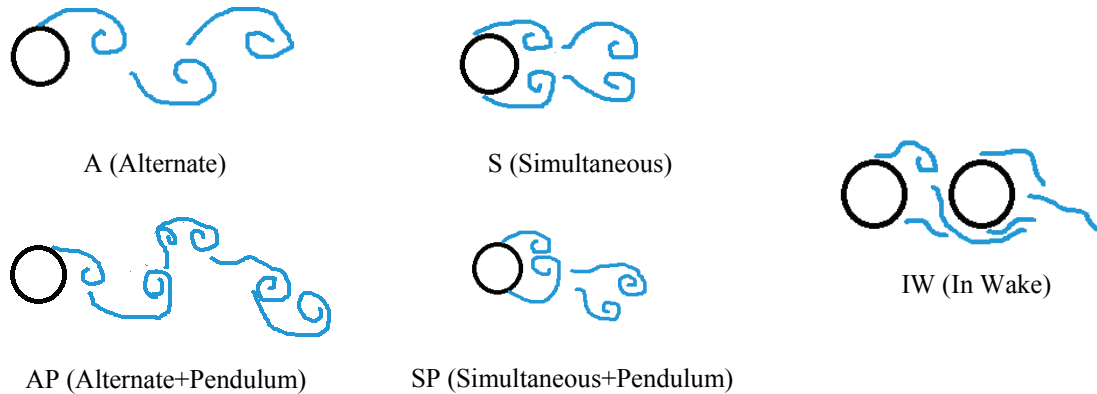


Fig. 5 The sketches of the vortex shedding situation from a circular cylinder

The aspects of vortex flow around oscillating tandem arranged circular cylinders are shown in Figs. 6~8. In each figure, the (a) figure shows the situation where the circular cylinders of tandem arrangement are not oscillating. The (b) figure and the (c) figure show the situation where the circular cylinders of tandem arrangement are oscillating in the situation where  $f_{VK}/f=1/2$  lock-in and  $f_{VK}/f=1/1$  lock-in occur in the case of a single circular cylinder, respectively. The flow patterns of distance ratio  $L/d=1.5$  are shown in Fig. 6. In the situation (see the (a) figure) where the circular cylinder is not oscillating, the situation where two circular cylinders perform the vortex shedding alternately as one cylinder is shown. However, when the circular cylinder was oscillated, aspects differed. The vortex shedding was performed respectively independently of the 1st circular cylinder and the 2nd circular cylinder. In the (b) figure, in spite of carrying out cylinder oscillation in the situation where  $f_{VK}/f=1/2$  lock-in occurs in the case of a single cylinder, by the near wake of the 1st circular cylinder, the separating shear layer has been rolled simultaneously and the twins vortex is formed. The 'S' situation was shown by the near wake of the 1st circular cylinder. On the other hand, the vortex shedding is carried out in the near wake of the 2nd circular cylinder, changing a direction alternately. The 'AP' situation is shown. And vortex arrangement of mushroom section form was seen in the far wake. In the (c) figure, by the near wake of the 1st circular cylinder, the separating shear layer has been rolled simultaneously and twin vortices are formed. With the 2nd circular cylinder, the vortices are shed simultaneously and the twin vortex street is formed. The flow patterns of distance ratio  $L/d=2.5$  are shown in Fig. 7. In the situation (see the (a) figure) where the circular cylinder is not oscillating, the separating shear layer of the 1st circular cylinder carries out a reattachment to the 2nd circular cylinder, and the situation of performing a vortex shedding alternately from the 2nd circular cylinder is shown. In the (b) figure, since cylinder oscillation is carried out in the situation of producing  $f_{VK}/f=1/2$  lock-in in the case of a single cylinder, while the 1st circular cylinder and the 2nd circular cylinder change a direction alternately, vortices are shed. The 'AP' situation is shown for both. In the (c) figure, since cylinder oscillation is carried out in the situation of producing  $f_{VK}/f=1/1$  lock-in in the case of a single cylinder, in the 1st circular cylinder and the 2nd circular cylinder, vortices are shed simultaneously. The situation of 'S' is shown for both. However, the natural Karman vortex street is formed, without carrying out the lock-in in the far wake of the 2nd circular cylinder. The flow patterns of distance ratio  $L/d=5.5$  are shown in Fig. 8. In the situation (see the (a) figure) where the cylinder is not oscillating, the separating shear layer of the 1st circular cylinder always forms a vortex alternately, and the situation of performing a vortex shedding alternately also from the 2nd circular cylinder is shown. In the (b) figure, since cylinder oscillation is carried out in the situation of producing  $f_{VK}/f=1/2$  lock-in in the case of a single cylinder, while both the 1st circular cylinder and the 2nd circular cylinder change a direction alternately, the vortices are shed simultaneously. The 'SP' situation is shown for both. In the (c) figure, since cylinder oscillation is carried out in the situation of producing  $f_{VK}/f=1/1$

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lock-in in the case of a single cylinder, by the near wake of the 1st circular cylinder, vortices are simultaneously shed from both sides of cylinder, and the twin vortex street is formed. Also the 2nd circular cylinder, vortices are simultaneously shed by the near wake. However, the twin vortex street is not formed of interference with the vortex discharged from the 1st circular cylinder. The alternate vortex street which has not produced the lock-in is formed.

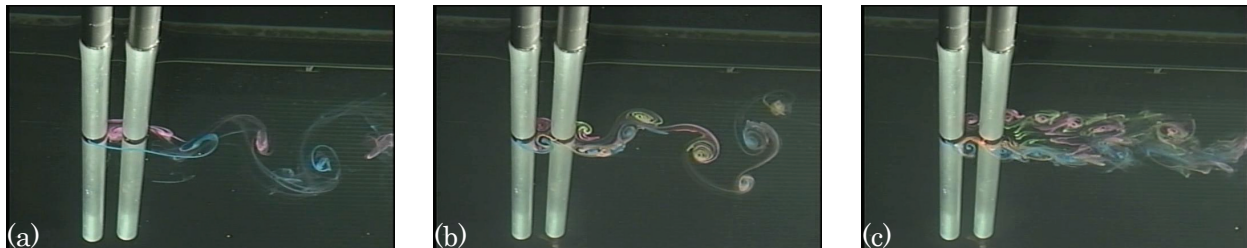


Fig. 6 Representative flow patterns in the case of distance ratio  $L/d=1.5$  and amplitude ratio  $2a/d=0.25$ , (a)  $f/f_K=0.00$ , (b)  $f/f_K=1.33$ , (c)  $f/f_K=2.39$

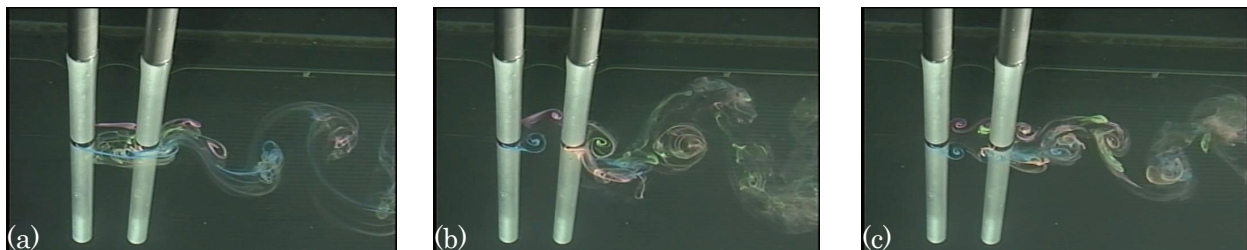


Fig. 7 Representative flow patterns in the case of distance ratio  $L/d=2.5$  and amplitude ratio  $2a/d=0.25$ , (a)  $f/f_K=0.00$ , (b)  $f/f_K=1.21$ , (c)  $f/f_K=2.40$



Fig. 8 Representative flow patterns in the case of distance ratio  $L/d=1.5$  and amplitude ratio  $2a/d=0.25$ , (a)  $f/f_K=0.00$ , (b)  $f/f_K=1.44$ , (c)  $f/f_K=2.98$

**Effect of Oscillation on Vortex Shedding;** The variations of mean vortex shedding frequency ratio  $f_{VK}/f_K$  with oscillation ratio  $f/f_K$  are shown in Figs.9~11. In the figure, the abscissa is the oscillation frequency ratio  $f/f_K$  and the ordinate is the vortex shedding frequency ratio  $f_{VK}/f_K$ . Two solid lines in the figure have the ratio of the vortex shedding frequency in case of the cylinder oscillation  $f_{VK}$  and the circular cylinder oscillating frequency  $f$  and mean the “lock-in” status. Therefore, the occurrence of lock-in can be assumed when experiment data are shown on the line. Here, the coloring symbols in the figure have classified the situation of the vortex shedding. Also in two circular cylinders of tandem arrangement, the lock-in was observed like the single circular cylinder. Even when cylinder oscillation frequency  $f$  is lower than the natural Karman vortex shedding frequency  $f_K$ , the lock-in phenomenon can be seen. It was obtained that the lock-in range of each of two circular cylinders of tandem arrangement is wider than the lock-in range of single cylinder case [3]. Figure 9 is a lock-in diagram in the case of the amplitude ratio  $2a/d=0.25$  in distance ratio  $L/d=1.5$ . The (a) figure shows the lock-in of the near wake of the 1st circular cylinder, and the (b) figure shows the lock-in of the near wake of the 2nd circular cylinder, respectively. And the (c) figure shows the lock-in of the far wake

of the 1st circular cylinder, and the (d) figure shows the lock-in of the far wake of the 2nd circular cylinder, respectively. In the 1st circular cylinder, it is almost  $f_{VK}/f=1/1$  lock-in in the near wake. On the other hand, with the 2nd circular cylinder, the change of a lock-in state as shown in the figure was seen. At the near wake and far wake of the 2nd circular cylinder, the difference was seen with larger cylinder oscillation frequency than oscillation frequency ratio  $f/f_k=2$ . In the case of  $2a/d=0.5$  which enlarged the oscillating amplitude ratio, the difference was not so much regarded as the time of amplitude ratio  $2a/d=0.25$ . Figure 10 is a lock-in diagram in the case of the amplitude ratio  $2a/d=0.25$  in distance ratio  $L/d=2.5$ . In the cylinder near wake, the change of the lock-in state where the 1st circular cylinder and the 2nd circular cylinder are the same was seen. At the far wake of the 2nd circular cylinder, it has swerved from the lock-in state with cylinder oscillation frequency about  $f/f_k=2$ . Although it seemed that the range of  $f_{VK}/f=1/2$  lock-in state of the 2nd circular cylinder was large a little at the time of oscillating amplitude ratio  $2a/d=0.5$ , the tendency of a change of a lock-in state was the same. Here, although distance is between circular cylinders to some extent, in the far wake of the 1st circular cylinder, there is no data all over a figure because it becomes the appearance by which the vortex shedding is not seemingly made in order that the separating shear layer discharged from the 1st circular cylinder may

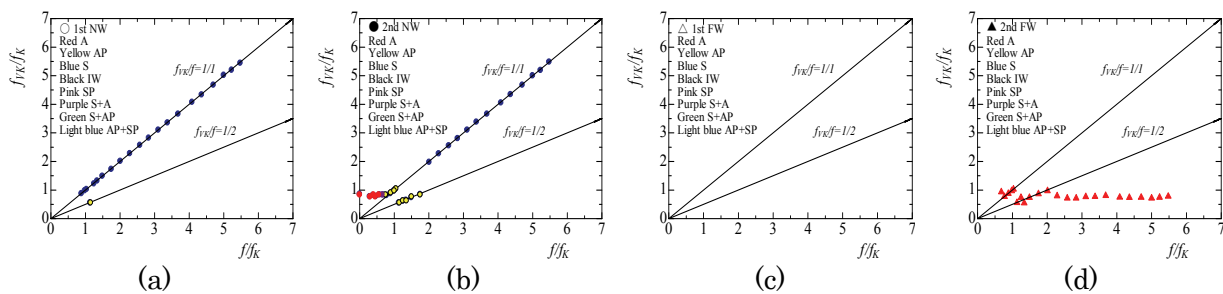


Fig. 9 Variation of mean vortex shedding frequency ratio in the case of distance ratio  $L/d=1.5$  and amplitude ratio  $2a/d=0.25$ , (a) 1st cylinder near wake, (b) 2nd cylinder near wake, (c) 1st cylinder far wake, (d) 2nd cylinder far wake

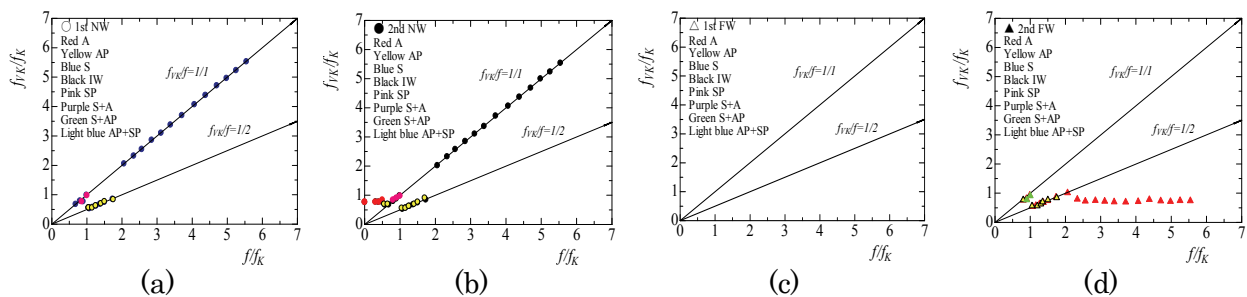


Fig. 10 Variation of mean vortex shedding frequency ratio in the case of distance ratio  $L/d=2.5$  and amplitude ratio  $2a/d=0.25$ , (a) 1st cylinder near wake, (b) 2nd cylinder near wake, (c) 1st cylinder far wake, (d) 2nd cylinder far wake

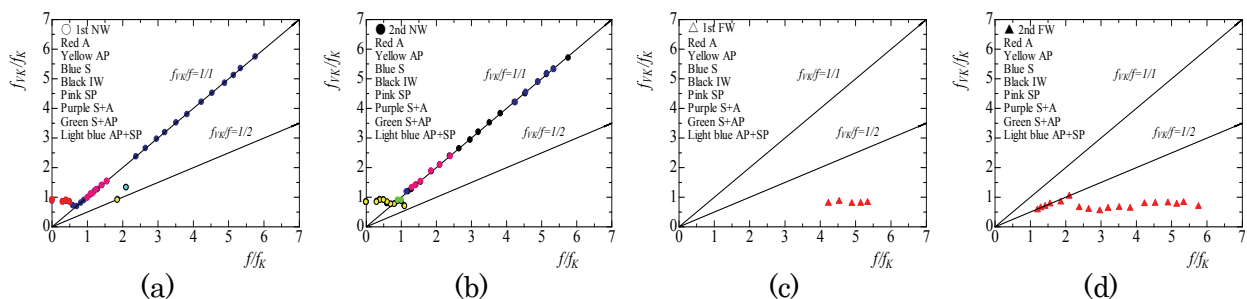


Fig. 11 Variation of mean vortex shedding frequency ratio in the case of distance ratio  $L/d=5.5$  and amplitude ratio  $2a/d=0.25$ , (a) 1st cylinder near wake, (b) 2nd cylinder near wake, (c) 1st cylinder far wake, (d) 2nd cylinder far wake

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wear on the 2nd circular cylinder. Figure 11 is a lock-in diagram in the case of the amplitude ratio  $2a/d=0.25$  in distance ratio  $L/d=5.5$ . In the near wake, although the circular cylinder interval is wide, it is shown for both of circular cylinders that most data is in  $f_{VK}/f=1/1$  lock-in state. And an alternate vortex shedding can be seen contained in the appearing flow pattern. By the far wake of the 1st circular cylinder, although a vortex street is observed, it is shown that it will not be in a lock-in state. It is shown by the far wake of the 2nd circular cylinder that it will be in  $f_{VK}/f=1/2$  lock-in state and that it will not be in a lock-in state. In the case of tandem cylinders, it was found in the far wake of the 2nd circular cylinder that a twin vortex street is not formed.

The region and configuration distribution which the lock-in in each distance ratio  $L/d$  produces are shown in Figs. 12~14. The abscissa is oscillation frequency ratio  $f/f_K$ , and ordinates are oscillating amplitude ratio  $2a/d$  in each figure. The single symbol in the figure means  $f_{VK}/f=1/1$  lock-in, and the duplication symbol means  $f_{VK}/f=1/2$  lock-in, and the data of the 1st circular cylinder is shown in the lower row, and it shows the data of the 2nd circular cylinder to the upper row, respectively. Moreover, the lock-in region in the case of single circular cylinder and configuration distribution are also shown for comparison. It can be seen that the oscillating frequency of the lock-in which is produced in the case of single circular cylinder is not necessarily applied in the case of oscillating tandem arranged two circular cylinders. It can be understood that the distribution situations of the range or a lock-in configuration which carry out a lock-in differ with the 1st circular cylinder and the 2nd circular cylinder. The difference was remarkable in especially the 2nd circular cylinder.

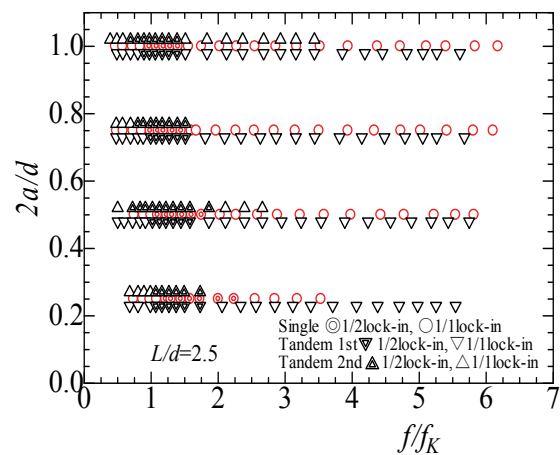
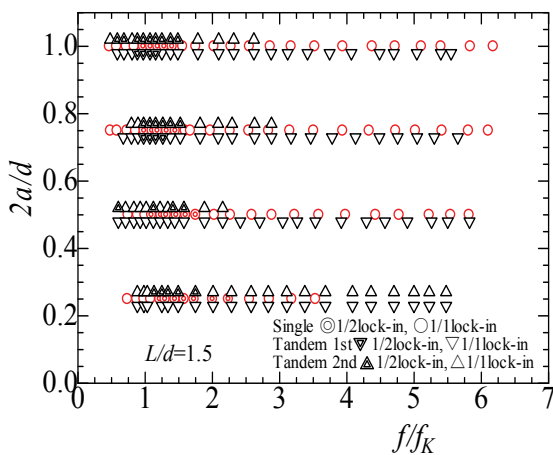


Fig. 12 Lock-in map in the case of distance ratio  $L/d=1.5$

Fig. 13 Lock-in map in the case of distance ratio  $L/d=2.5$

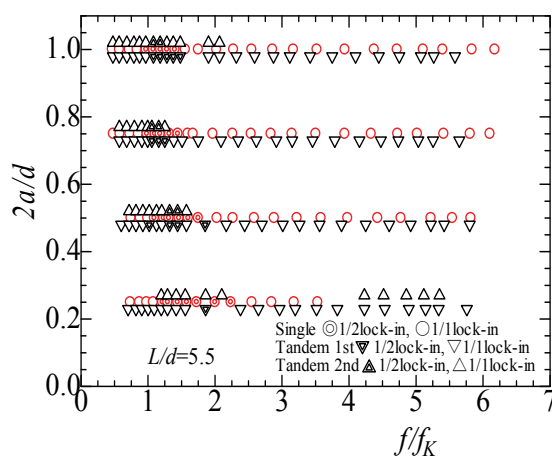


Fig. 14 Lock-in map in the case of distance ratio  $L/d=5.5$

**CONCLUSIONS.** The features of vortex shedding from tandem arranged circular cylinders oscillating to the direction of the flow were observed with the aid of the laser light sheet technique. The following conclusions were obtained.

- (1) The typical flow patterns of the lock-in or un-lock-in states were shown every distance ratio  $L/d$ .
- (2) In distance ratio  $L/d=2.5$  and  $L/d=5.5$ , the twin vortex street from the 2nd circular cylinder was not seen.
- (3) Although the cylinder oscillation frequency  $f$  is lower than the natural Karman vortex's frequency  $f_K$ , the lock-in phenomenon can be seen.
- (4) The lock-in ranges have been shown in each distance ratio. And the lock-in ranges become wide when comparing with the single cylinder case.
- (5) The lock-in region and the lock-in range are differing with the 1st circular cylinder and the 2nd circular cylinder, respectively.

### Nomenclature

- $a$  half amplitude of cylinder motion, m  
 $d$  outside diameter of cylinder, m  
 $f$  cylinder oscillation frequency, Hz  
 $f_K$  natural Karman vortex shedding frequency, Hz  
 $f_{VK}$  vortex shedding frequency, Hz  
 $L$  cylinder distance, m  
 $Re$  Reynolds number, non-dimensional  
 $U$  main flow velocity, m/s

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