Propagation of a Gaussian Laser Beam with Elliptical Cross-section through a Medium in the Presence of a Standing Acoustical Wave

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Abstract

In the present paper the effect of refractive index variations due to a standing acoustic wave on the Gaussian laser beam with an elliptical cross-section will be investigated. The effects related to the spatial boundedness of the beam are studied i.e. distortions of the complex amplitude of the beam resulting from interference of the spatial spectrum components are analyzed. The long propagation distances of the laser beam or high sound frequencies are on consideration. To solve the problem, we represent the beam field as a spatial spectrum and describe propagation of each spectral component in the inhomogeneous medium in the approximation of geometrical optics using methods of eikonal and amplitude perturbation. At the observation point, the optical field represents a superposition of partial waves whose interference (with allowance for the perturbations) gives rise to laser beam amplitude and phase distortions. Using the concept of geometrical-optics rays for partial waves, we impose the corresponding limitation on distance that the laser beam travels in the media. Using the example of a Gaussian beam with an elliptical cross-section, we analyzed distortions of this beam in the presence of a standing acoustic wave.

Introduction

For investigation of highly convective flows in gasdynamic mixing lasers [1] and corresponding acoustic fields may be used the known methods of flow diagnostics [2]. Application of laser Doppler anemometry (LDA) [2] to determining parameters of the acoustic field in a liquid requires the analysis of the laser beam distortions that occur in a medium in the presence of an acoustic wave. In [3-4] were the phases distortions of optical field in LDA investigated that may be observed when two coherent laser beams intersect and create interference fringes at the point of interest. The distortions of the envelope of the laser beam intensity in the presence of a traveling or standing acoustic wave were in [5,6] considered.

Methodology and computer simulation

Fig.1. shows the geometric parameters of the problem on consideration.

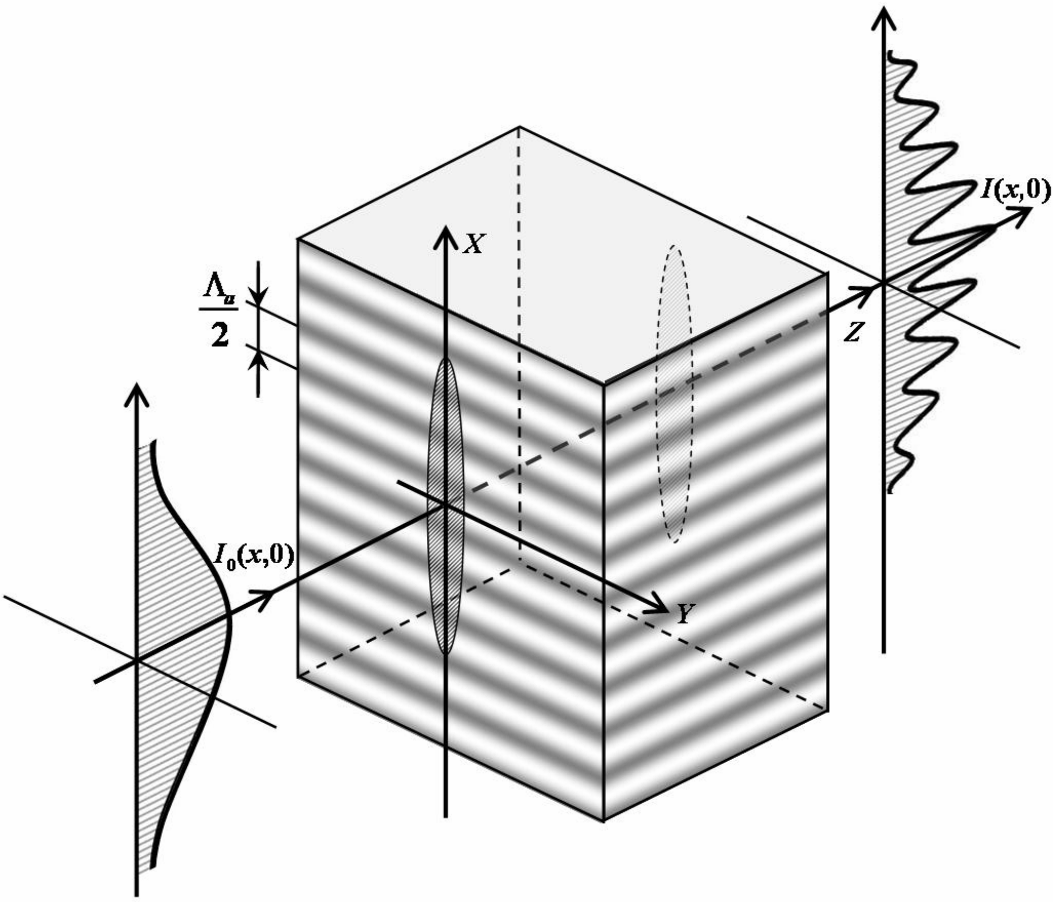


Fig. 1. The geometric illustration of the mathematical model.

A medium with standing acoustic wave located at $z \geq 0$. A laser beam with elliptical cross section and two effective sizes w_1 (x-axis direction) and w_2 (y-axis direction) propagates in this medium. There is laser sheet when $w_1 << w_2$ or $w_2 << w_1$. Parameters $\boldsymbol{K_a}$ and $\boldsymbol{k}$ are the acoustic and optical wave vectors, where its absolute values are respectively:

$$K_a = \frac{2\pi}{\Lambda_a}, \qquad k = \frac{2\pi}{\lambda}, \tag{1}$$

and Λ_a and λ are the acoustic and optical wavelengths in the medium. The beam axis makes angle α with the Z axis at $z = 0$. Below we assume that $\sin\alpha << 1$. Let the complex amplitude of the laser beam at $z = 0$ (at the entrance to the medium) be $E(x,y,0)$:

$$E(x, y, 0) = \exp\{i[kx \sin\alpha]\} A^0(x, y, 0), \tag{2}$$

where $A^0(x, y, z)$ is the beam envelope in the medium without acoustic wave

$$A^0(x, y, z) = A_x^0(x, z) A_y^0(y, z) = A^0 \exp\left\{ -\frac{x^2}{w_1^2} - \frac{y^2}{w_2^2} \right\}, \tag{3}$$

where $A_x^0(x, z)$, $A_y^0(y, z)$ are corresponding x and y depended components.

The refractive index of the medium modulated by the acoustic wave can be represented

$$n(x,t) = n_0(1 + 2\delta n \sin(\Omega_a t)\cos(K_a x)), \qquad \delta n = \frac{\Delta n}{n_0} << 1, \tag{4}$$

where Ω_a - the acoustic frequency, t – time, n_0 - the refractive index of the unperturbed medium and Δn - the maximum deviation from n_0. The beam field at the point of observation with coordinates (x, y) calculated using the expression derived in [5] for x-depended refractive index:

$$E(x, y, z, t) = \frac{\exp\{i[kz\cos\alpha + kx\sin\alpha + \Delta\varphi_0(x, z, t)]\}}{\sqrt{\gamma(x, z, t)}} A_x^0(x + \Delta x(x, z, t), z + \Delta z(x, z, t)) A_y^0(y, z), \tag{5}$$

where the phase perturbation $\Delta\varphi_0$ corresponds to the phase screen approximation with a stable beam envelope shape. The value γ describes the amplitude perturbation because the convergence and divergence of geometric rays inside the beam and calculated by use of transport equations. The value Δx describes the effects caused by interference of spatial spectrum components and Δz – the diffraction effects in inhomogeneous media. The specified values calculated at defined parameters of acoustic field (see /5/). The beam intensity envelope is

$$|A|^2 = \frac{\left| A_x^0(x+\Delta x(x,z,t), z+\Delta z(x,z,t)) A_y^0(y,z) \right|^2}{\gamma(x,z,t)}. \tag{6}$$

For the standing acoustic wave at α≠0 the expressions for specified functions are:

$$\Delta\varphi_0(x,z,t) = \frac{2\delta n \cdot k}{K_a \sin\alpha}((1-\cos\zeta)\sin K_a x + \sin\zeta \sin K_a x)\sin\Omega_a t, \tag{7}$$

$$\Delta x(x,z,t) = \frac{2\delta n \cdot z}{\sin\alpha}\left(\frac{\zeta\sin\zeta-(1-\cos\zeta)}{\zeta}\sin K_a x + \frac{\zeta\cos\zeta-\sin\zeta}{\zeta}\cos K_a x\right)\sin\Omega_a t, \tag{8}$$

$$\Delta z(x,z,t) = \frac{2\delta n \cdot K_a z^2}{\sin\alpha}\times$$

$$\times\left(\frac{\zeta^2\cos\zeta-2\zeta\sin\zeta+2(1-\cos\zeta)}{\zeta^2}\sin K_a x - \frac{\zeta^2\sin\zeta+2\zeta\cos\zeta-2\sin\zeta}{\zeta^2}\cos K_a x\right)\sin\Omega_a t, \tag{9}$$

In addition, using the perturbation method, we have found an explicit expression for ray trajectories in a weakly heterogeneous medium whose refractive index is modulated by a standing acoustic wave and, based on the transport equation, we took bulk effects into account.

$$x'(z,x_0) = x - 2\frac{\delta n \cdot z}{\sin\alpha}\left(\frac{\zeta\sin\zeta-(1-\cos\zeta)}{\zeta}\sin K_a x + \frac{\zeta\cos\zeta-\sin\zeta}{\zeta}\cos K_a x\right)\sin\Omega_a t. \tag{10}$$

$$\gamma(x,y,z) = \sqrt{1+2\delta n\frac{K_a z}{\sin\alpha}\sin\Omega_a t\left(\frac{\zeta\cos\zeta-\sin\zeta}{\zeta}\sin K_a x - \frac{\zeta\sin\zeta-(1-\cos\zeta)}{\zeta}\cos K_a x\right)}, \tag{11}$$

where

$$\zeta = K_a z \cdot tg\alpha. \tag{12}$$

At α=0 we use corresponding limits of these functions, for example:

$$\Delta x(x,y,z) = \delta n K_a z^2 \sin K_a x \sin\Omega_a t, \tag{13}$$

$$\gamma(x,y,z) = \sqrt{1-\delta n (K_a z)^2 \cos K_a x \sin\Omega_a t}. \tag{14}$$

Fig.2-7 demonstrate the dependence of the beam intensity envelope on distances z and time instances t (parameter $T = \Omega_a t/ 2\pi$) for different values w_1 , w_2 and at $\alpha\neq0$ or $\alpha=0$. Fig. 2,a,b demonstrate the refraction of laser sheet in x – direction. The corresponding to $\delta n=4\cdot10^{-5}$ acoustic pressure in water is about 3.5 atmosphere. The maximal shift of the beam axis at the distance z=100 mm is approximately equal 0.5 mm. This effect related to the spatial boundedness of the beam and refraction shift of the envelope treating as distortions of the complex amplitude of the beam results from interference of the spatial spectrum components.

Fig.3,a-f ($\alpha\neq0$) and Fig.4,a-d ($\alpha=0$) show the variation of beam intensity envelope in the media with acoustic wave at different distances z for the case when $w_1\approx\Lambda_a$ (Λ_a=1mm, w_1=1 mm, w_2=0.1 mm), T=0.25, $\delta n=3\cdot10^{-5}$. The intensity envelope distortions caused by convergence (maximum on the graphic) and divergence (minimum on the graphic) of geometrical rays inside the laser beam take place. Using the concept of geometrical-optics rays, we impose the corresponding limitation on distance that the beam travels in the media [5]. At beam focusing distances (Fig. 4,d) we may use the

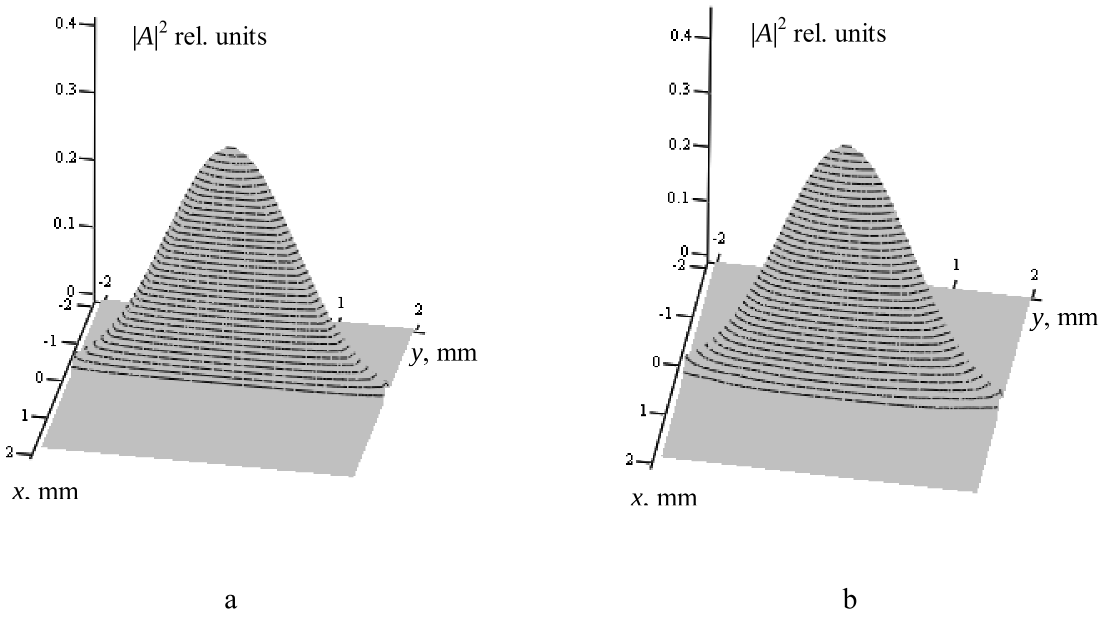


Fig. 2. The envelope of beam intensity at two different instances t (a – T=0.4; b – T=0.7) for the case when $w_1 \ll \Lambda_a$ (Λ_a=1 mm, w_1=0.05 mm, w_2=1 mm) and at $\sin\alpha=0.05$, z=100 mm, $\delta n=4\cdot10^{-5}$.

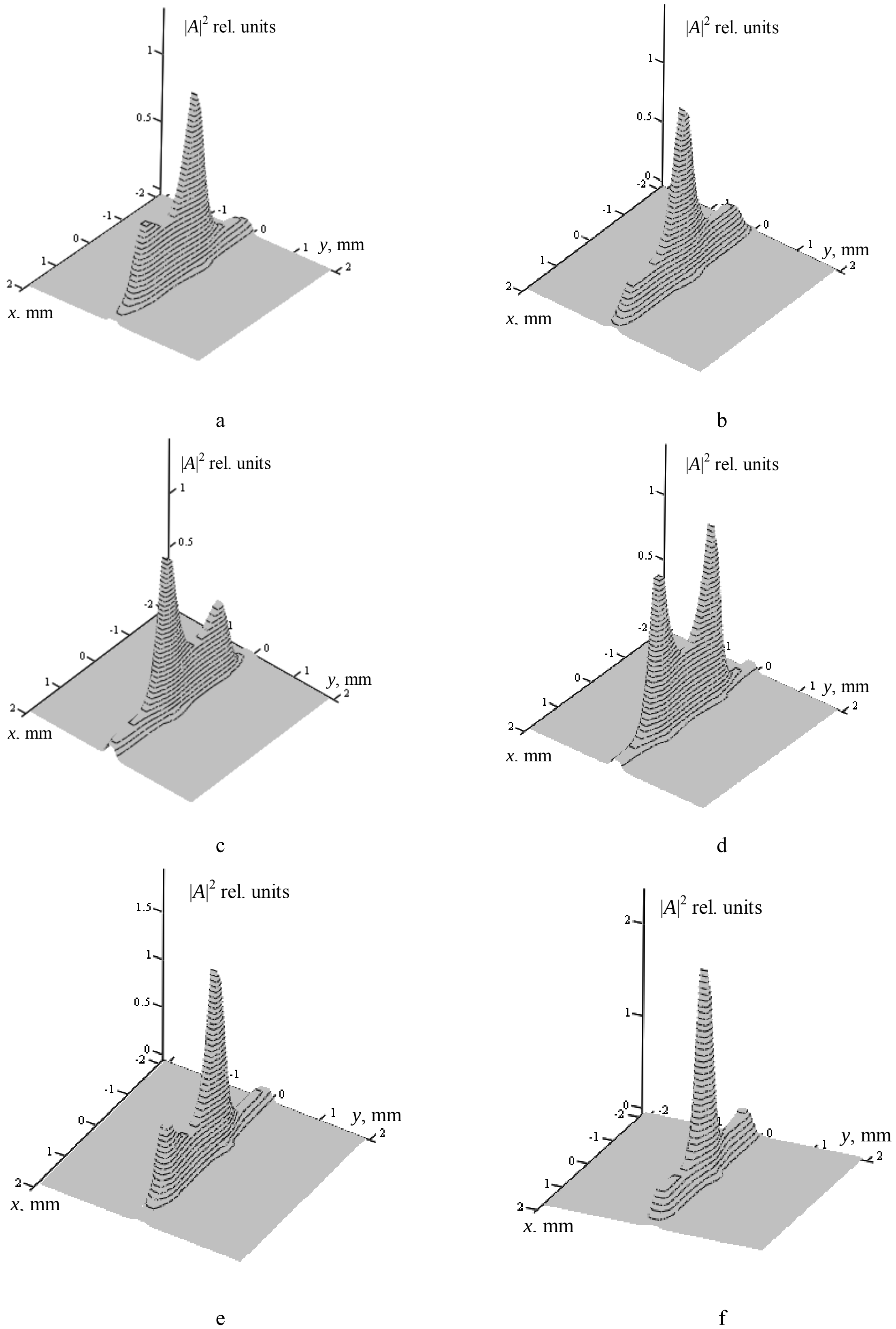


Fig. 3. The beam intensity envelope in the media with acoustic wave for the case when $w_1 \approx \Lambda_a$ (Λ_a=1 mm, w_1=1 mm, w_2=0.1 mm), T=0.25, $\sin\alpha$=0.05, $\delta n = 3\cdot 10^{-5}$ at different distances z: a – 100 mm, b – 105 mm, c – 110 mm, d – 115 mm, e – 120 mm, f – 125 mm.

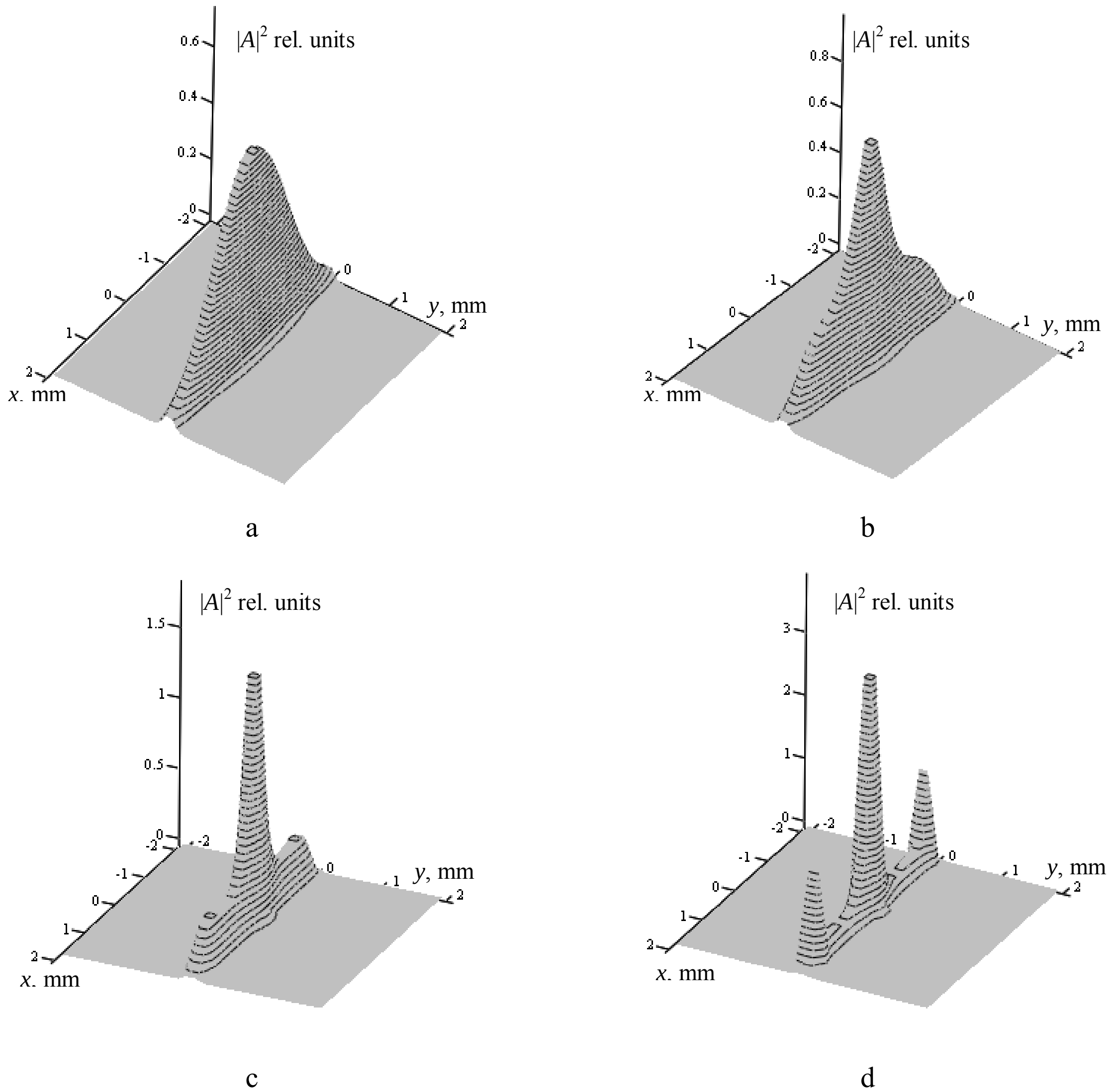


Fig. 4. The beam intensity envelope in the media with acoustic wave for the case when $w_1 \approx \Lambda_a$ (Λ_a=1 mm, w_1=1 mm, w_2=0.1 mm), T=0.25, sinα=0, $\delta n = 3 \cdot 10^{-5}$ at different distances z: a – z=0 mm, b – z=20 mm, c – z=27 mm, d – z=29 mm.

modification of geometrical-optics. When α≠0 and the acoustic fields are weak enough, there is not beam focusing along its trajectory in the media. In the case α≠0 the envelope is essentially asymmetrical. Fig.5. demonstrates the variation of beam intensity envelope in the media with acoustic wave for the case when $w_1 \approx \Lambda_a$ (Λ_a=1 mm, w_1=1 mm, w_2=0.1 mm), z=25 mm, sinα=0, $\delta n = 3 \cdot 10^{-5}$ at different values of T during the period of acoustic oscillation. The case $w_1 \approx \Lambda_a$ corresponds to the situation when the envelope has the most distortions. In the next case $w_1 >> \Lambda_a$, Λ_a=0.15 mm, w_1=1 mm, w_2=0.1 mm (see Fig.6,a-f) the envelope is modulated by acoustic field and its variations are

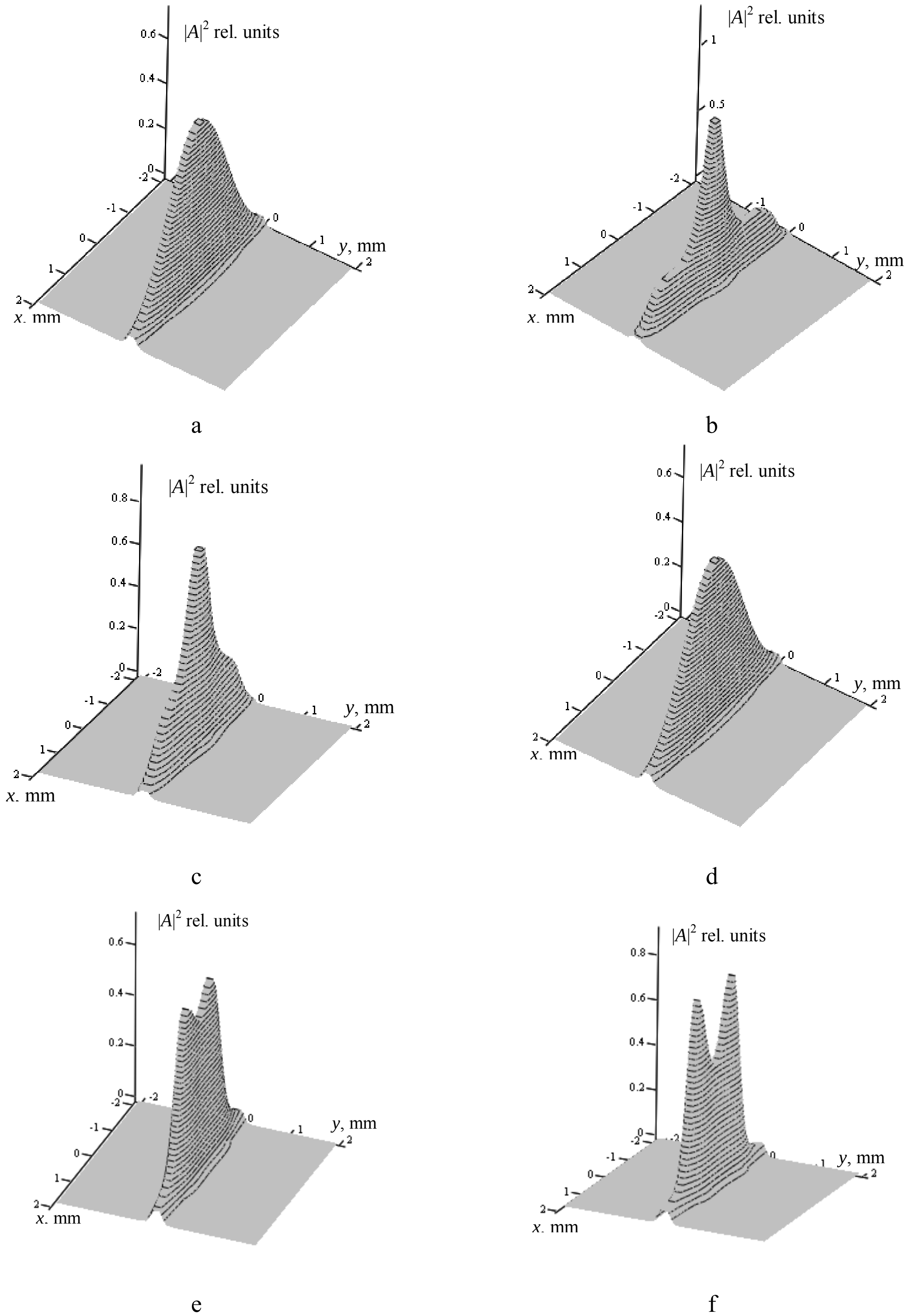


Fig. 5. The beam intensity envelope in the media with acoustic wave for the case when $w_1 \approx \Lambda_a$ (Λ_a=1 mm, w_1=1 mm, w_2=0.1 mm), z=25 mm, sinα=0, $\delta n = 3 \cdot 10^{-5}$ at different values of T: a – T=0, b – T= 0.2, c – T=0.4, d – T=0.5, e – T=0.6, f – T=0.8.

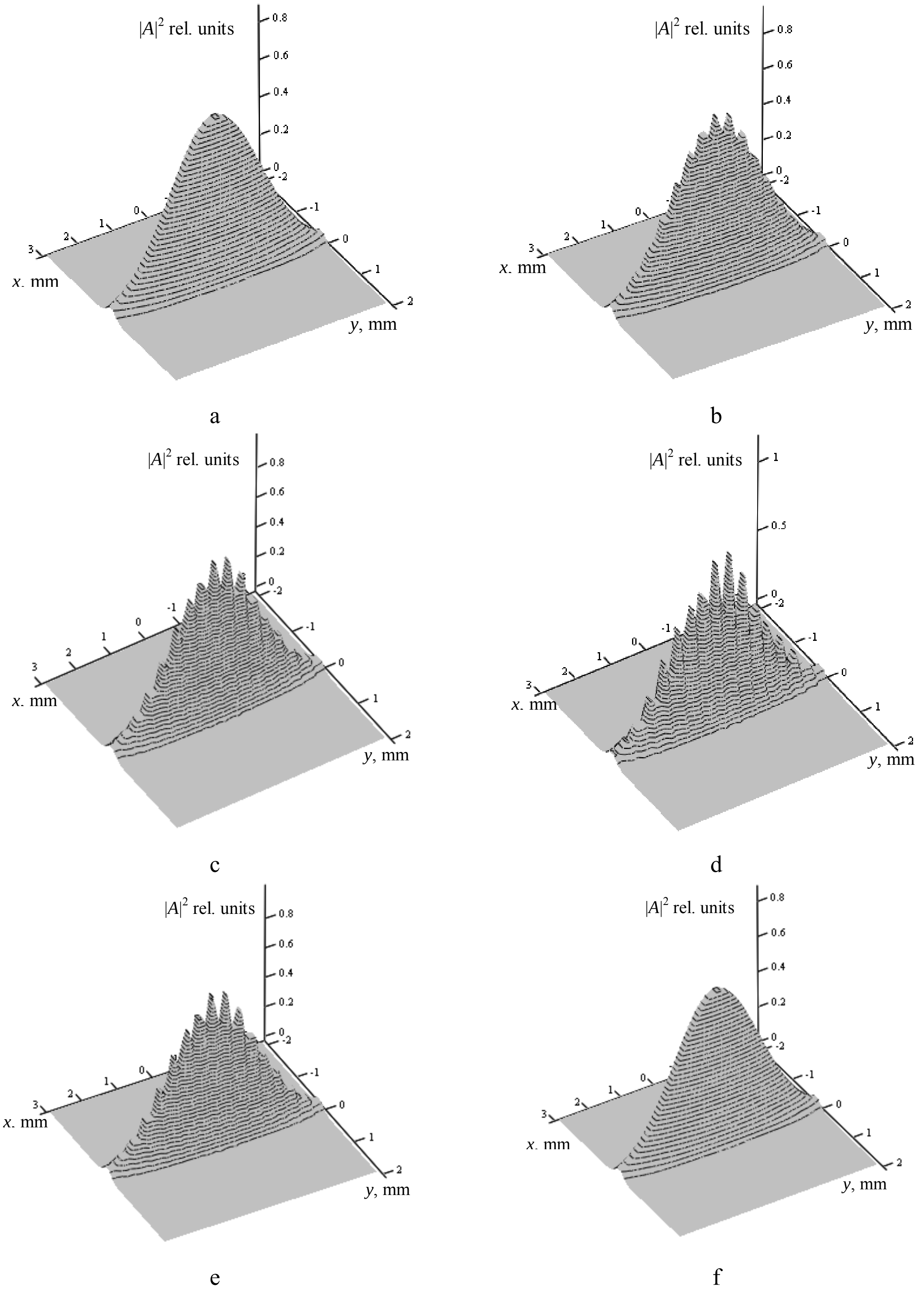
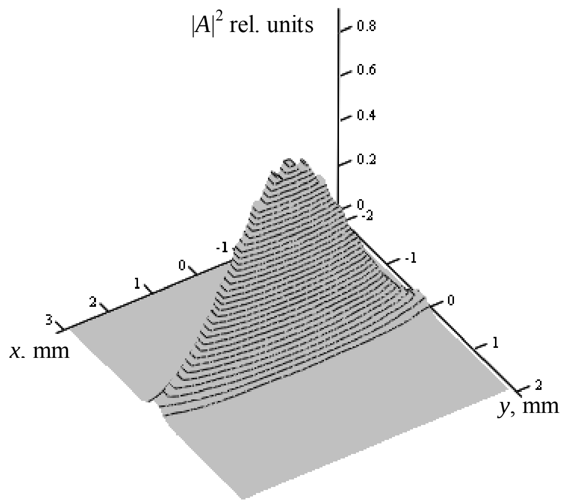
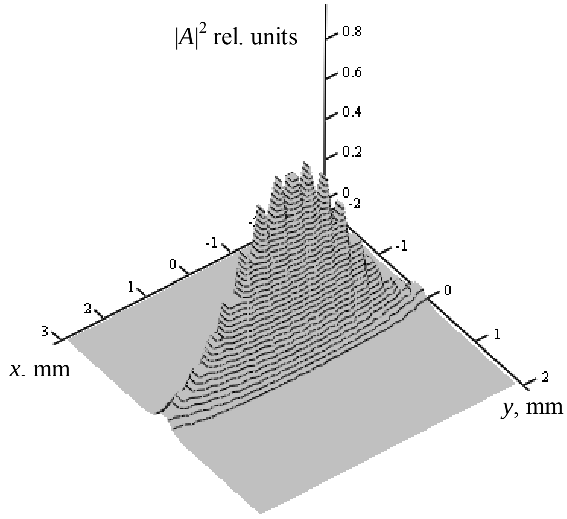


Fig. 6. The beam intensity envelope in the media with acoustic wave for the case when $w_1 >> \Lambda_a$ (Λ_a=0.15 mm, w_1=1 mm, w_2=0.1 mm), z=10 mm, sinα=0.05, $\delta n = 3\cdot 10^{-5}$ at different values of T: a – T= 0, b – T=0.05, c – T=0.1, d – T=0.25, e – T=0.4, f – T=0.5.

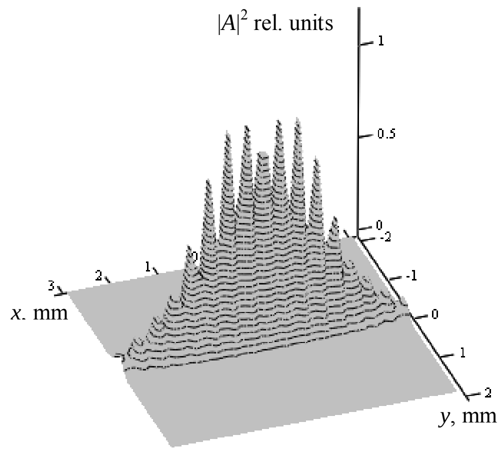
shown during the period of acoustic oscillation. Fig.7, a-d (α=0) show the variation of beam intensity envelope for the case when $w_1 >> \Lambda_a$ (Λ_a=0.15 mm, w_1=1 mm, w_2=0.1 mm), T=0.25, sinα=0, $\delta n=10^{-5}$ at different distances z. The situation is similar to the case Fig.4, a-d considered above.



a



b



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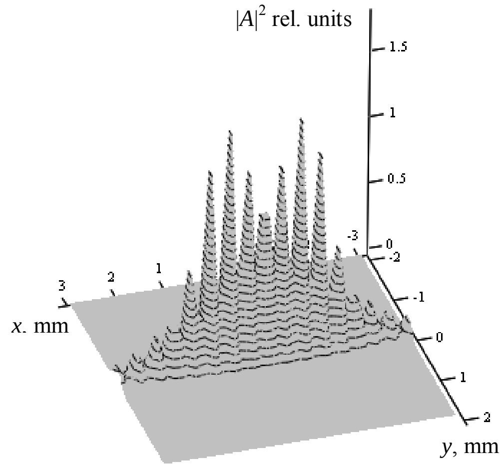

d

Fig.7. The beam intensity envelope in the media with acoustic wave for the case when $w_1 >> \Lambda_a$ (Λ_a=0.15 mm, w_1=1 mm, w_2=0.1 mm), T=0.25, sinα=0, $\delta n=10^{-5}$ at different distances z: a –z=2 mm, b – z=4 mm, c – z=6 mm, d – z=7 mm.

Conclusion

Analytical expressions describing distortions of the complex amplitude of a laser beam propagating in a medium with a weak perturbation of the refractive index caused by an acoustic wave are derived. A distinctive feature of the expressions for the optical field is that they are written in the same form for a wide range of ratios of the acoustic wavelength to the beam sizes. This ratio only affects the character of field distortions. Using the example of a Gaussian beam with an elliptical cross-section, we analyzed distortions of this beam in the presence of a standing acoustic wave and demonstrated that, for the acoustic wavelength less then the beam radius the results obtained correspond to the Raman--Nath solution. For the beam radius less then the acoustic wavelength, the results coincide with the solution to the refraction problem. Based on the latter fact, we interpreted the refractive displacement of the beam envelope at a given distance as a particular case of amplitude distortion. The formulas derived reveal common roots of refraction and diffraction effects. Thus, the expressions obtained make it possible to study an intermediate scenario when the beam radius approximately equal to the acoustic wavelength.

The practical value of the results obtained lies in the possibility of estimating the error in local parameters of acoustic fields measured using optical methods and determining the applicability limits of these methods. In addition, the phase and amplitude of optical radiation which has passed through a weakly heterogeneous medium are functions of its integral parameters. Therefore, analytical expressions for the beam field serve as a basis for solving the inverse problem and enable one to develop integral methods for laser diagnostics of acoustic fields. The results of calculation of the Gaussian laser beam intensity in the presence of the acoustic wave are the simulation of the quantitative visualization of acoustic field as a phase object.

Acknowledgements

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