Pseudo-Steady Shock Wave Reflections: A State-of-the Knowledge Review

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Abstract

The distinguished philosopher Ernst Mach published the first known paper on the phenomenon of planar shock wave reflections over straight ramps over 125 years ago in 1878 [1]. In his publication he presented two wave configurations that could result from this reflection process, namely a regular reflection (RR) and a configuration that was later named after him and was called Mach reflection (MR). In 1945, Smith [2] reported on an additional wave configuration, which was slightly different from the Mach reflection wave configuration that was reported by Mach. Smith [2] did not ascribe any special importance to the wave configuration that he observed. The wave configuration was recognized as an independent one only about 5 years later when White [3] published the discovery of a new wave configuration, that was named as double-Mach reflections (DMR) because it had similar features to that of the Mach reflection wave configuration that was discovered by Mach [1] but all the features repeated in it twice. For this reason the Mach reflection wave configuration has been re-named to single-Mach reflection. The discovery of the double-Mach reflection revealed that the wave configuration that was first observed by Smith [2] was an intermediate wave configuration between the single-Mach reflection (**S**MR) and the double-Mach reflection (DMR) wave configurations. For this reason it was named transitional-Mach reflection (TMR).

Since the discovery of the DMR many investigations were aimed at elucidating the exact transition criteria between the above-mentioned four different wave configurations as well as some additional sub-configurations that were discovered later.

In 1991 Ben-Dor [4] published a monograph, entitled "Shock Wave Reflection Phenomena", that was, in fact, a state-of-the-knowledge review of the phenomena.

A few years later, in 1995, Li & Ben-Dor [5] modified the analytical approach for evaluating the transition criteria from the single-Mach to the transitional-Mach reflection ($\text{SMR} \leftrightarrow \text{TMR}$) and from the transitional-Mach to the double-Mach reflection ($\text{TMR} \leftrightarrow \text{DMR}$), and presented some new criteria for the formation and termination of both the TMR and DMR wave configurations.

Experimental results from various sources revealed that the transition boundaries between the SMR, TMR and DMR wave configurations that were based on the modified analytical approach were indeed more accurate than those that were summarized in the Ben-Dor's monograph [4].

Unfortunately, however, the results of the modified analytical approach of Li & Ben-Dor [5] have not been internalized, and publications by various authors in the past decade neglected the revised and better transition criteria, and kept on referring to the wrong criteria that appear in Ben-Dor's monograph [4]. For this reason, the above-mentioned 10-year old work of Li & Ben-Dor [5] is presented again.

Introduction

When a planar shock wave that propagates with a constant velocity encounters a sharp compressive straight planar ramp in a shock tube, it interacts with the ramp surface and reflects over it.

Depending upon the shock wave Mach number, M_s, and the reflecting ramp angle, θ_w, the resulted wave configuration can be one of the following two general types: a regular reflection, RR, or an irregular reflection, IR, wave configuration.

The IR can be either a von-Neumann reflection, vNR, which is typical to small ramp angles and weak shock waves or a Mach reflection-MR.

The Mach reflection wave configuration, which in pseudo-steady flows is always a direct-Mach reflection, i.e., a Mach reflection in which its triple point, T, moves away from the reflecting surface [see e.g., Courant & Friedrichs [6], can be further divided into three different types: a single-Mach reflection, SMR, a transitional-Mach reflection, TMR, and a double-Mach reflection, DMR. Schematic illustrations of the wave configurations of these five types of reflection are shown in figure 1. Mach [1] was the first to observe and report on the RR and the SMR wave configurations, Smith [2] discovered the TMR wave configuration, White [3] discovered the DMR wave configuration, and the vNR was first reported slightly over a decade ago by Colella & Henderson [7].

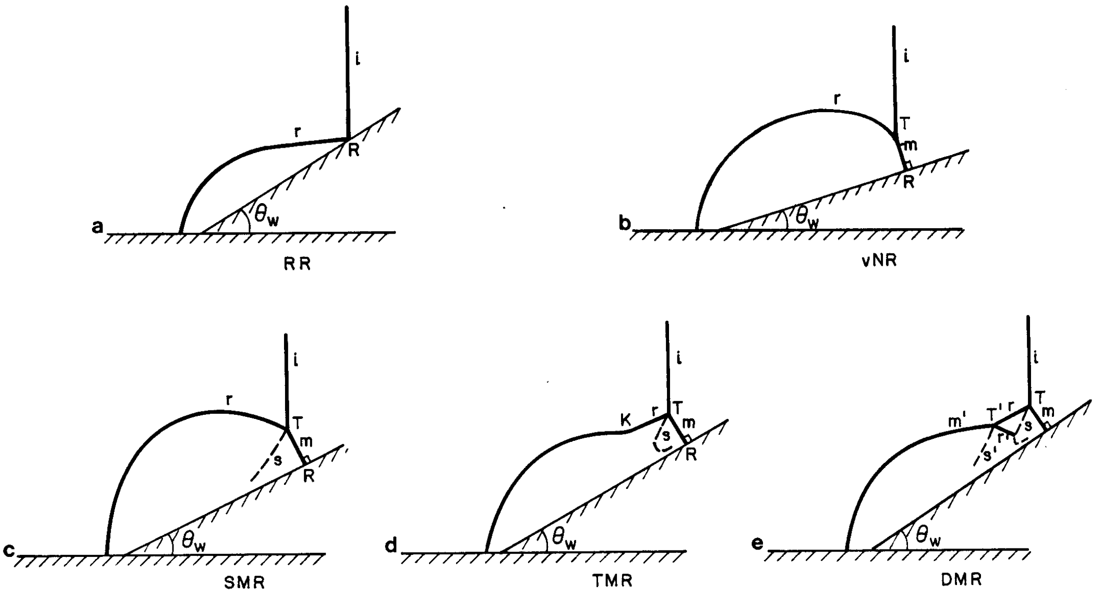

Figure 1: Schematic illustrations of the wave configurations of a) regular reflection, RR, b) von Neumann reflection, vNR, c) single-Mach reflection, SMR, d) transitional-Mach reflection, TMR, and e) double-Mach reflection, DMR.

The Wave Configurations and Their Transition Criteria: State-of-the-Knowledge

Li & Ben-Dor [5] showed that the best approach to derive the transition criteria between the above presented shock wave reflection configurations is to consider the entire interaction process to be a combination of two sub-processes, namely:

- The *shock wave reflection process*, i.e., the reflection of the incident shock wave over the reflecting surface, and
- The *flow deflection process*, i.e., the deflection of the incident shock wave induced flow around the leading edge of the reflecting ramp.

Law [8] referred to the interaction of these two processes as the *shock diffraction process*. This approach, which was initiated more than 30 years ago by Law [8] and Law & Glass [9] and followed soon after by Ben-Dor [10], was, in fact, ignored by almost all the researchers who searched for the transition criteria between the above-mentioned various wave configurations. Instead, they limited their investigations to deriving the various transition criteria by considering the shock wave reflection process only.

Li & Ben-Dor [5] showed that the transition criteria that were derived by considering the shock reflection process only and neglecting the flow deflection process and its interaction with the shock wave reflection process failed in accurately separating between the above-mentioned different wave configurations. They also showed that wave configurations of both the TMR and the DMR are, in fact, dominated by the intensity of the interaction between these two processes, and as a consequence their domains and transition criteria cannot be determined unless the interaction process between these two processes is accounted for.

Modes of interaction between the two processes

Consider figure 2 in which the above-mentioned shock wave reflection and the flow deflection processes are schematically illustrated. The shock wave reflection is an MR with a triple point, T, and a reflected shock wave, r, which is seen to extend to point Q. The flow state behind the reflected shock wave is denoted by (2). For the reader's convenience the slipstream and the notation of the other flow regions of the MR wave configuration are not shown in figure 2. The incident shock wave Mach number is M_s and the shock-induced flow Mach number is M_1^L. The bow shock wave (B), which arises from the deflection process of the shock wave induced flow around the leading edge of the ramp, extends up to point b. The flow domain between point b to which the bow shock, B, extends and point Q to which the reflected shock wave, r, extends is the domain in which the interaction between the two processes takes place.

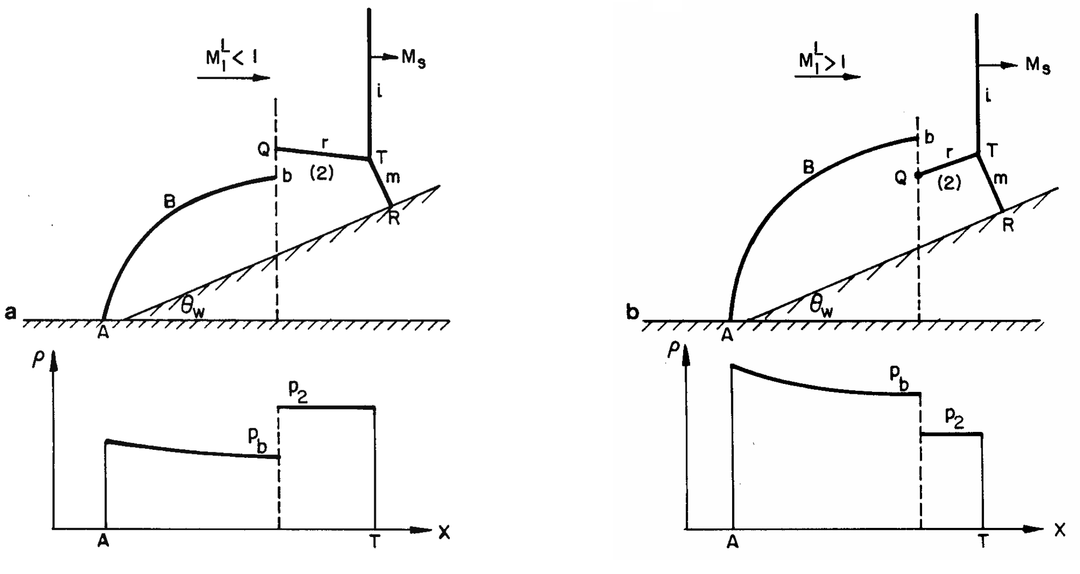

Figure 2: Schematic illustration of two possible interaction mechanisms between the shock wave reflection and the flow deflection processes. a) The incident shock induced flow is subsonic, $M_1^L < 1$, and b) the incident shock induced flow is supersonic, $M_1^L > 1$.

Let us define the pressure behind the reflected shock wave p_2 and that behind the bow shock wave p_b. Li & Ben-Dor [5] showed that these two pressures, i.e., p_2 and p_b, can be used to express the "strength" of the shock wave reflection and the flow deflection processes, respectively, and that their magnitudes determine the mechanism by which the flow field that results from the interaction of these two processes are patched.

- If $p_b < p_2$ (see figure 2a) then a band of expansion disturbances propagate towards region (2) behind the reflected shock wave, r, to bridge the pressure difference $p_b - p_2 < 0$, and
- If $p_b > p_2$ (see figure 2b) then a band of compression disturbances propagate towards region (2) behind the reflected shock wave, r, to bridge the pressure difference $p_b - p_2 > 0$.

Based on Semenov & Syshchikova [11] the boundary between these two situations, i.e., $p_b = p_2$, is associated with the situation in which the incident shock-induced flow is exactly sonic, i.e., $M_1^L = 1$. As a consequence,

- $p_b < p_2$ when $M_1^L < 1$, and
- $p_b > p_2$ when $M_1^L > 1$

as is indicated in figures 2a and 2b, respectively.

The transition criteria

The $RR \leftrightarrow IR$ transition criterion

Out of the various suggested criteria for the $RR \leftrightarrow IR$ transition, i.e., the termination of the RR wave configuration and the formation of the IR wave configuration (see Ben-Dor [4] Sec. 1.5, for details), the one that best agrees with pseudo-steady shock-tube experimental data, is the one arising from the length scale concept that was forwarded by Hornung et al. [12]. Following their concept, the RR wave configuration terminates, in pseudo-steady flows, when the flow behind the reflected shock wave, r, of the reflection point, R, becomes sonic in a frame of reference attached to point R. This criterion implies that the $RR \leftrightarrow IR$ transition occurs when

$$M_2^R = 1 \qquad (1)$$

where M_2^R is the flow Mach number in state (2), behind the reflected shock wave of the RR wave configuration, with respect to the reflection point R. As long as $M_2^R > 1$, the corner-generated signals, which result from the flow deflection process around the leading edge of the ramp, cannot catch-up with the reflection point, R, and an IR wave configuration, which is typified by a finite length shock wave that is known as the Mach stem, is impossible. The condition $M_2^R > 1$ implies that a physical length scale is unavailable at the reflection point, R, and as a result the reflected shock wave, r, is straight in the vicinity of the reflection point.

The $MR \leftrightarrow vNR$ transition criterion

Once the corner-generated signals have caught up with the reflection point, R, an IR wave configuration, typified by a finite length shock wave that is known as the Mach stem, is formed. Note that the above-mentioned communication of a physical length scale to the reflection point, R, is a condition for the formation of wave configurations containing a finite length shock wave like the IR wave configurations.

As mentioned earlier the IR wave configuration can be either an MR wave configuration or a vNR wave configuration. The parameter determining whether the IR wave configuration is a vNR or an MR is the angle of incidence, ϕ_2, between the direction of the flow in state (1), behind the incident shock wave, i, and the reflected shock wave, r.

The reflection is an MR wave configuration as long as $\phi_2 < \pi/2$. Consequently, the $MR \leftrightarrow vNR$ transition takes place when

$$\phi_2 = \frac{\pi}{2} \tag{2a}$$

When $\phi_2 = \pi/2$, the flow passing through the reflected shock wave is perpendicular to the shock wave. Consequently, the flow is not deflected by the reflected shock wave and it remains perpendicular to the reflected shock wave. However, based on the boundary conditions across the slipstream, s, the flow behind the reflected shock wave must be parallel to the slipstream. Consequently, it is obvious that the condition given by (2a) can be rewritten as

$$\omega_{rs} = \frac{\pi}{2} \tag{2b}$$

where ω_{rs} is the angle between the reflected shock wave, r, and the slipstream, s. It should be noted here that the above-mentioned reflected shock wave, which is clearly visible in the MR wave configuration degenerates to a band of compression waves in the vNR wave configuration.

The SMR $\leftrightarrow$ TMR & DMR transition criteria

Once the just-mentioned condition for the existence of an MR wave configuration is met the value of the flow Mach number, in state (2), behind the reflected shock wave of the MR, with respect to the triple point T, i.e., M_2^T, becomes the significant parameter that determines the particular type of the obtained Mach reflection wave configuration.

As long as $M_2^T < 1$ the wave configuration is that of an SMR typified by a reflected shock wave, which is curved along its entire length. The fact that the reflected shock wave is curved along its entire length implies, as mentioned earlier, that a physical length scale is communicated through state (2) to the triple point from which the curved reflected shock wave emanates. Gasdynamic considerations imply that this communication path is possible only as long as the flow in state (2) is subsonic with respect to the triple point, T, i.e., $M_2^T < 1$.

When the flow in state (2) becomes supersonic with respect to the triple point, T, i.e., $M_2^T > 1$, a supersonic flow zone blocks the just-mentioned communication path and the reflected shock wave develops a straight portion that is terminated by a kink, K, which most probably indicates the point along the reflected shock wave that has been reached by the leading-edge-generated signals, i.e., the point where the interaction between the shock wave reflection and the flow deflection processes takes place. Thus the SMR wave configuration terminates and gives rise to either a TMR or a DMR wave configurations when

$$M_2^T = 1 \tag{3a}$$

Shirouzu & Glass [13] proposed an additional necessary (but not sufficient) condition for the termination of the SMR and the formation of the TMR and the DMR wave configurations, which has the property of slightly shifting the transition line based on the criterion given by (3a). Their additional condition was based on the assumption of Law & Glass [9] that the horizontal velocity of the kink, K, is equal to the incident shock wave induced flow velocity. Based on this assumption, they concluded that in an SMR wave configuration $\omega_{ir} < \pi/2$. Here ω_{ir} is the angle between the incident and the reflected shock waves. Based on this experimental finding, which has not been supported as yet by any analytical explanation, the following condition should also be met in order to enable the termination of the SMR and the formation of either the TMR or the DMR wave configurations

$$\omega_{ir} = \frac{\pi}{2} \qquad (3b)$$

Once the earlier-mentioned kink has been formed along the reflected shock wave, r, of the MR wave configuration, the value of the flow Mach number in state (2) behind the reflected shock wave with respect to the kink, K, becomes the significant parameter in determining whether the resulted wave configuration is a TMR or a DMR.

Based on the earlier remark that the wave system, by which the flow fields that result from the shock reflection and flow deflection processes are patched, depends on whether p_b is greater or smaller than p_2, Li & Ben-Dor [5] proposed to distinguish between two cases that lead to different types of wave configurations:

Case 1: when $p_b < p_2$, the resulted wave configuration is a pseudo-TMR in which the leading disturbance propagating towards region (2) is an expansive wave. As a result, no reversal of curvature exists along the reflected shock wave, r, of the TMR wave configuration. For this reason it is termed pseudo-TMR.

Case 2: when $p_b > p_2$, the resulted wave configuration is either a TMR or a DMR. The specific type of the resulted wave configuration depends on whether the interaction between the shock reflection and the flow deflection processes is "weak" or "strong".

- The interaction is weak if the leading disturbance propagating towards region (2) is a compressive wave, and
- The interaction is strong if the leading disturbance propagating towards region (2) is a shock wave that is formed from the convergence of the just-mentioned compression waves.

The intensity of the interaction, i.e., whether it is weak or strong, is determined by the incident shock wave Mach number, M_s, and the reflecting ramp angle, θ_w. Li & Ben-Dor [5] showed that if the interaction is weak, i.e., when a distributed band of compression waves propagates towards region (2) and the resulted wave configuration is a TMR, the leading disturbance of this band of compression waves, which does not converge to form a shock wave, interacts with the reflected shock wave, r, at the kink, K[1], and forces it to reverse its curvature. Thus M_2^K is always equal to unity in the case of a TMR wave configuration. Li & Ben-Dor [5] presented an analytical model for predicting the location of the kink of a TMR wave configuration.

However, if the interaction is strong, the compression waves converge to form a shock wave, r'. This shock wave forces the reflected shock wave, r, to develop a strong discontinuity (a sharp kink) that develops to become the second triple point, T'. Based on gas dynamic considerations [6], a secondary slipstream must compliment this triple point.

Li & Ben-Dor [5] presented two simplified analytical models that described two different DMR wave configurations for determining the location of the second triple point T' of the DMR wave configuration.

[1] Note that in the case of a TMR wave configuration the kink is not as sharp as in the case of a DMR wave configuration. It is in fact a point along the reflected shock wave where a reversal of curvature takes place.

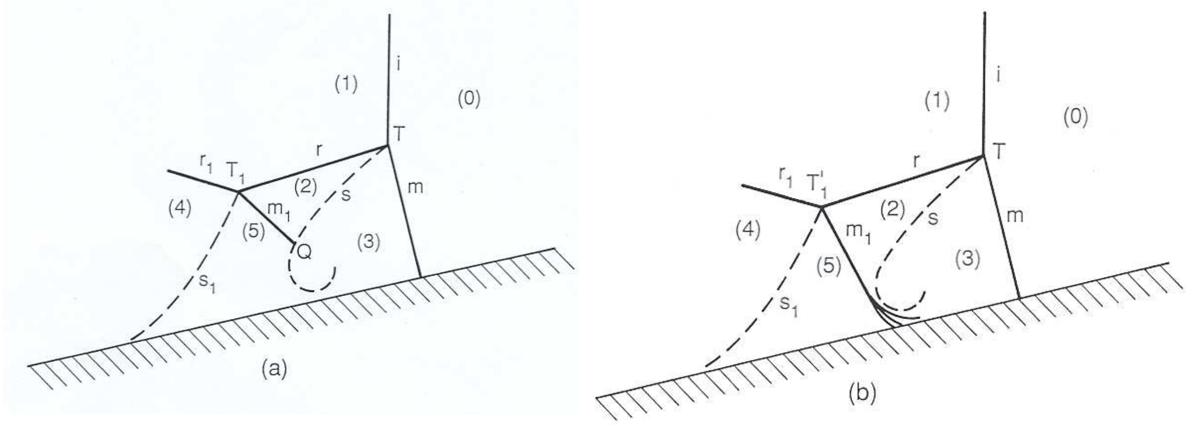

Figure 3: Schematic illustration of two different possible wave configurations of double Mach reflection a) the secondary reflected shock wave terminates perpendicularly on the primary slipstream, and b) the secondary reflected shock wave terminates on the reflecting wedge surface at the point where the primary slipstream reaches the reflecting surface.

The just-mentioned two DMR wave configurations, which are schematically illustrated in figures 3a and 3b, differ in the way the secondary reflected shock wave, r', interacts with the primary slipstream, s.

- In the first case, shown in figure 3a, the secondary reflected shock wave, r', terminates perpendicularly somewhere along the primary slipstream, s.
- In the second case, shown in figure 3b, the secondary reflected shock wave, r', terminates at (or near) the point where the primary slipstream, s reaches the reflecting ramp surface. Gasdynamic considerations [6] imply that in this case r' does not have to be perpendicular to s.

The analytical models that were developed by Li & Ben-Dor [5] enables one to accurately calculate the location of the second triple point, T', along the reflected shock wave, r. The accurate location of the second triple point is needed in order to transform the frame of reference from the first to the second triple point. This transformation is needed in order to calculate the flow Mach number in state (2) behind the reflected shock wave with respect to the second triple point, $M_2^{T'}$ since the condition for the existence of the DMR wave configuration is $M_2^{T'} > 1$.

Li & Ben-Dor [5] proved, based on simple gasdynamic and physical considerations [6], that under no circumstances could the flow Mach number behind the second triple point become supersonic with respect to the second triple point, $M_4^{T'}$, i.e., $M_4^{T'} < 1$ always! Consequently, the hypothesis of Ben-Dor [4] and Ben-Dor & Takayama [14] that additional wave configurations, e.g., a triple-Mach reflection, can occur is wrong, and only the above presented five wave configurations, i.e., RR, vNR, SMR, TMR and DMR are possible in the case of the reflection of a planar incident shock wave over a sharp compressive straight planar ramp surface.

It should be mentioned here that Vasilev & Kraiko [15] showed numerically that the wave configuration that results in the weak shock wave domain is not a vNR but a configuration which was first predicted by Guderley [16] almost 60 years ago. Skews & Ashworth [17] claimed recently that they managed to experimentally verify Vasilev & Kraiko's [15] finding and showed Guderley's [16] prediction. The wave configuration that was predicted by Guderley [16] consisted of a Prandtl-Meyer expansion fan immediately behind the reflected shock wave. Skews & Ashworth [17] suggested to name this wave configuration after Guderley and call it Guderley reflection, GR.

Based on the forgoing presentation the conditions and requirements for the transitions between SMR, TMR and DMR are as follows. The necessary and sufficient conditions for the $\mathrm{SMR} \leftrightarrow \mathrm{TMR}$ transition are

$$M_2^T = 1 \quad (4a)$$

$$M_1^L = 1 \quad (4b)$$

where, as mentioned earlier, M_2^T is the flow Mach number in region (2) in a frame of reference attached to the first triple point, T, and M_1^L is the incident shock-wave induced flow Mach number, i.e., the flow Mach number in region (1) in the laboratory frame of reference.

Determining the conditions that sharply distinguish between the transitional and the double-Mach reflections is much more difficult. This is due to the fact that the TMR wave configuration, as its name indicates, is a primary stage of the DMR wave interaction. Consequently, these two wave configurations are compatible and distinguishing between them is sometimes impossible.

In general, it can be said that the condition for the existence of the TMR wave configuration is

$$M_2^K = 1 \quad (5)$$

Similarly, the condition for the existence of the DMR wave configuration is

$$M_2^{T'} > 1 \quad (6)$$

It should be noted here that as shown by Li & Ben-Dor [5] the kink K of the TMR wave configuration and the second triple point T' of the DMR wave configuration are two different points whose locations along the reflected shock wave, r, are calculated using different analytical models.

Ben-Dor [4] showed analytically that depending on the relative values of the first, χ, and the second, χ', triple point trajectory angles, the DMR wave configuration could be divided into two subtypes; a positive DMR wave configuration, DMR^+, for which $\chi' > \chi$, and a negative DMR wave configuration, DMR^-, for which $\chi' < \chi$. Consequently, the $DMR^+ \leftrightarrow DMR^-$ occurs at

$$\chi' = \chi \quad (7)$$

Figure 4 presents the domains of the RR, the SMR, the TMR and the DMR wave configurations in the (M_s, θ_w)-plane for air.

- The SMR-wave configuration domain is labeled by A.
- The TMR-wave configuration domain is labeled by B.

Based on the foregoing discussion $M_2^K = 1$ everywhere inside this domain. The line separating domains A and B is given by equation (4a), i.e., $M_2^T = 1$.

The DMR-wave configuration domain is labeled by C. The line separating domains B and C is given by a slight modification of equation (6), i.e., $M_2^{T'} = 1 + \varepsilon$ where $\varepsilon \to 0$. The reason for not attempting to calculate a line for which $M_2^{T'} = 1$ lies in the fact that such a requirement implies that the secondary reflected shock wave, r', is, in fact, not a shock wave. Consequently, in order to ensure that r' remains a shock wave the condition $M_2^{T'} = 1 + \varepsilon$ should be used. The exact location of the line separating the TMR and the DMR wave configuration domains depends on the value chosen for ε. The value $\varepsilon = 0.01$ was used in the calculation shown in figure 4. Larger values of ε would shift the transition line further into the DMR wave configuration domain.

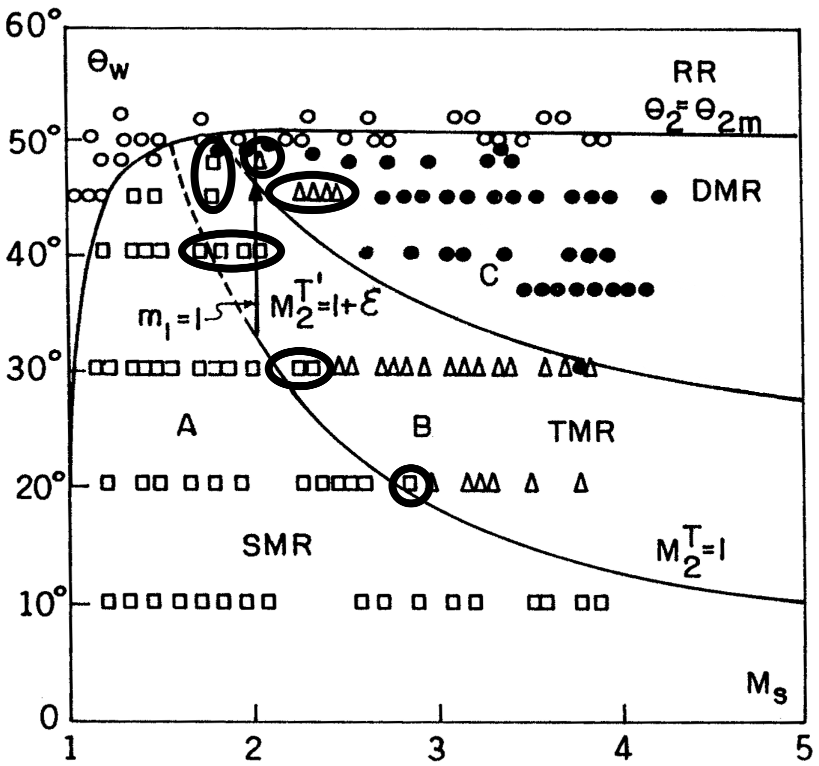

Figure 4: Verification of the transition lines as calculated using the state-of-the-knowledge criteria with experimental results (The experiments are taken from [16])

Note that since the existence of TMR and DMR wave configurations implies that the shock induced flow should be supersonic [see equation (4b)] the transition lines $M_2^T = 1$ and $M_2^{T'} = 1 + \varepsilon$ are terminated at $M_s = 2.07$, which is the value appropriate to $M_1^L = 1$.

The addition of this modification caused 5 SMR-experiments (squares), which previously lied inside the TMR-wave configuration domain, to be in their correct SMR-domain.

Summary

Five different wave configurations that can be obtained when a planar incident shock wave that propagates with a constant velocity reflects over the surface of a sharp compressive straight planar ramp in a shock tube were presented.

The five wave configurations are: the regular reflection, RR, the von Neumann reflection, vNR, the single-Mach reflection, SMR, the transitional-Mach reflection, TMR, and the double-Mach reflection, DMR. The DMR wave configuration can be subdivided into two wave configurations, the positive DMR, i.e., DMR^+ and the negative DMR, i.e., DMR^-.

The state-of-the-knowledge of the conditions for the formation, existence and termination of each one of these wave configurations, as well as their domains in the (M_s, θ_w)-plane were presented.

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