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Generalized method of the partial characteristics for description of radiation transfer in high-temperature gas dynamics

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Abstract

Radiation energy transfer in plasma is investigated with account for its actual optical properties. Consideration is based on the spectrum-integral method of partial characteristics. Databanks on integral partial characteristics of a number of substances are composed on the basis of data on optical properties that include the molecular state of matter, weakly ionized plasma, and plasma with multicharge ions and the main processes that determine absorption of thermal radiation (bound-bound, free-free, and bound-free transitions in molecules, atoms, and ions). This databank is based on the use of continuous (linear) splines for the description of spatial distribution of parameters of environment, as well as of discontinuous splines. Calculations of plasma radiation by this method are compared with results of the spectral description.

1. Introduction

Including radiation greatly complicates solution of problems of high-temperature gas dynamics [1, 2]. A radiant flux and its divergence are spectrum-integral and angular-integral quantities that depend on the distribution of the parameters (temperature, density) in the entire volume under consideration. They should be determined rapidly and with sufficient accuracy on each time layer. It is obvious that direct integration over the spectrum in solving a gas dynamics problem is not used due to its laboriousness. The multigroup approximation frequently used for simplification of calculations, where the actual spectrum is replaced by several tens of spectral groups within the limits of which the coefficients of absorption are averaged by one or another method (Rosseland methods or Planck mean-group methods, etc.), though giving a qualitatively accurate result, may greatly differ quantitatively from an approximation allowing for the actual spectrum. In particular, in [3] it is shown that under conditions where the radiation heat transfer is close to the limit of radiation thermal conductivity, the flow calculated in the multigroup approximation is several times lower than the actual one.

In the present paper, we used the spectrum-integral method of partial characteristics, which was developed in [4, 5 |, to describe radiation transfer in an actual spectrum. This method is developed for description of nonlinear wave motion in plasma. The method is based on representation of the radiant flux (for a plane layer) or the intensity in a prescribed direction (for an arbitrary geometry of the plasma) and their divergence in terms of spectrum-integral quantities. A databank of partial characteristics is composed for air, silicon dioxide, carbon dioxide, hydrogen, carbon, and aluminum. Detailed test calculations are made, and possible ways of accelerating calculations of a radiation field are studied.

2. Optical properties

To describe radiation transfer of energy we need data on the optical properties of matter. In the state of local thermal equilibrium they were calculated with account for the main radiation processes characteristic of hot gases and plasma. At temperatures $T \leq 10^4$ K, among these are absorption of radiation in eleclron-vibrational transitions in molecules and molecular ions, discrete transitions in atoms and ions, processes of photodissociation and photoionization of molecules, and free-free transitions in the fields of neutral particles and ions. As the temperature increases, discrete transitions in atoms and ions, processes of photoionization of ground arid excited levels and internal electron shells, and bremsstrahlung absorption in the fields of ions become predominant. A variety of radiation transitions, whose contribution to the total absorption changes sharply depending on the parameters of the plasma and the quantum energy, lead to a complex structure of the actual spectrum, which should be taken into account in solving problems of radiation transfer. In particular, radiation in spectral lines, in spite of their small width, can, in a number of cases, determine the magnitude of the radiant flux. This results in the need to describe the spectrum of the coefficient of absorption in detail.

Basic information on composed databanks of optic characteristics of matter is given in [6–13]. The databank on optical properties of matter contains a set of tables of mass coefficients of absorption $K_\varepsilon(T,\rho)=\kappa_\varepsilon(T,\rho)/\rho$ (cm^2/g) as a function of the energy of the quanta, the temperature, and the density. Mass coefficients of absorption depend much more weakly on the density than linear ones, which increases the accuracy of interpolation of the tables to the required values of the parameters of the substance. Figure 1 presents the spectral mass coefficient of absorption of air at four temperatures as an example illustrating the obtained data. Figure 2 shows absorption of aluminum plasma.

The figs. 3–4 illustrate the dependences of $K_\varepsilon(T,\rho)$ for air and carbon plasma on energy of quantum, temperatures and density. The typical spectral interval of averaging was equal to about $3\cdot10^{-2}$ eV. These data were used for calculation of partial characteristics.

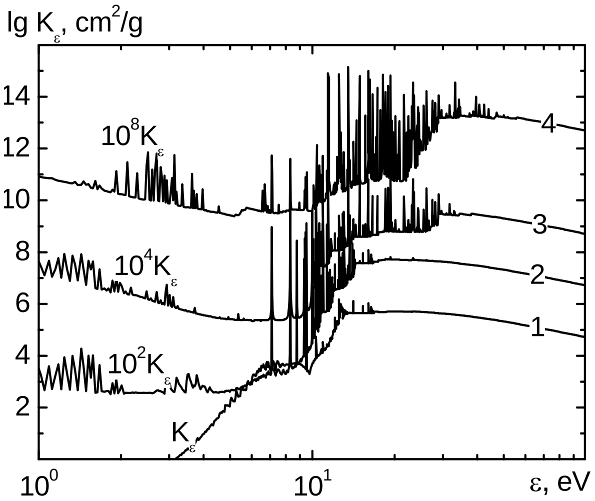

Fig. 1 Spectral absorption coefficients of air plasma at P=1 atm. 1– $T=5\times10^3$ K; 2–10^4; 3– 2×10^4; 4– 3.16 eV

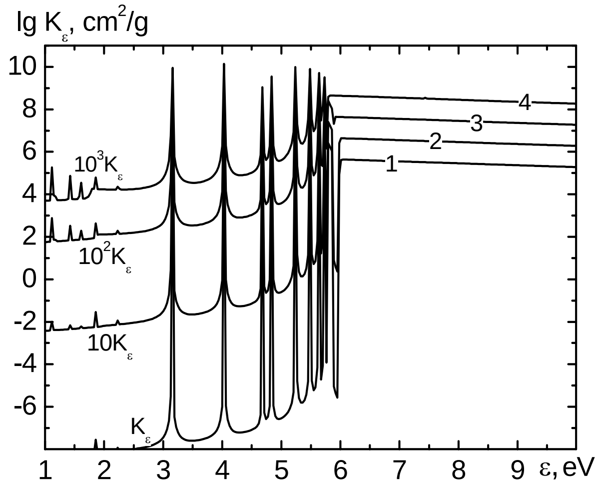

Fig. 2 Spectral absorption coefficients of aluminum at $\rho=1.29\times10^{-3}$ g/cm^3. 1 – T=0.1, 2 – 0.158, 3 – 0.251, 4 – 0.316 eV

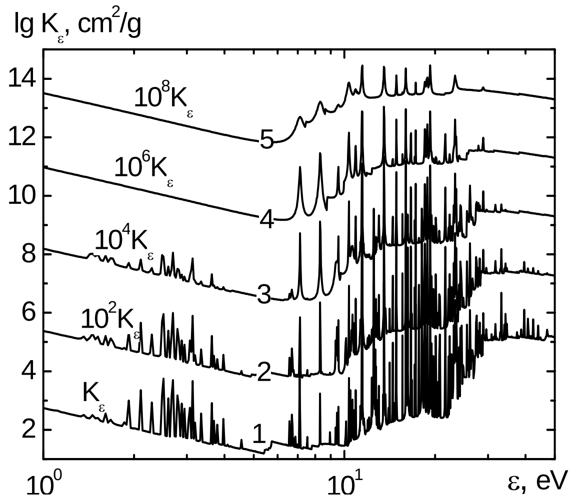

Fig. 3 Absorption coefficients of air plasma at $T=2.9\times10^4$ K. 1 – ρ=1.29×10^{-6} g/cm^3; 2 – 10 ρ; 3 – $10^2\,\rho$; 4 – $10^3\,\rho$; 5 – $10^4\,\rho$

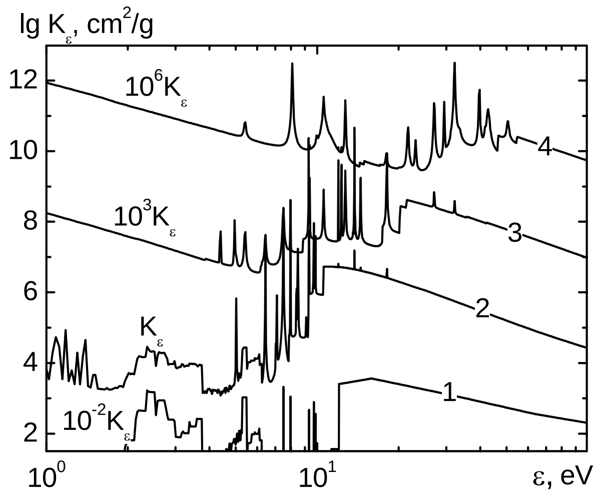

Fig. 4 Absorption coefficients of carbon plasma at ρ= 5.35×10^{-4} g/cm^3. 1 – T=3.5×10^3 K; 2 – 10^4K; 3 – 3.16 eV; 4–10 eV

3. Method of partial characteristics

Radiant flux and its divergence are integrated over a spectrum and an angular variable. These values depend on the distribution of parameters of substance in all considered volume and are determined by the equation of radiation transfer:

$$\overline{\Omega}\nabla I_\varepsilon = k_\varepsilon(B_\varepsilon - I_\varepsilon), \qquad \overline{F}(r) = \int_{(\varepsilon)} d\varepsilon \int_{(\Omega)} I_\varepsilon(r,\overline{\Omega})\overline{\Omega}d\Omega, \quad \mathrm{div}\overline{F} = \int_{(\varepsilon)} d\varepsilon\left[4\pi k_\varepsilon B_\varepsilon - \int_{(\Omega)} I_\varepsilon d\Omega\right] \tag{1}$$

The formal solution of equation (1) on a ray of length L in the absence of sources on the boundaries is determined by the expression

$$I(x) = \int_0^\infty\int_0^L B_\varepsilon(\xi)\kappa_\varepsilon(\xi)\exp\left(-\left|\int_\xi^x \kappa_\varepsilon(y)dy\right|\right)\mathrm{Sign(x\text{-}\xi)}\,d\varepsilon\,d\xi = \int_0^L \Delta I(\xi,x)\,\mathrm{Sign}(x-\xi)\,d\xi \tag{2}$$

$$\Delta I(\xi,x) = \int_0^\infty B_\varepsilon(\xi)\kappa_\varepsilon(\xi)\exp\left[-\left|\tau_\varepsilon(\xi,x)\right|\right]d\varepsilon \tag{3}$$

In the case of the plane layer when equation (1) is precisely integrated over the angular coordinate, we have

$$F(x) = 2\pi\int_0^\infty\int_0^L B_\varepsilon(\xi)\kappa_\varepsilon(\xi)\,E_2\left(\left|\int_\xi^x \kappa_\varepsilon(y)dy\right|\right)\mathrm{Sign(x\text{-}\xi)}\,d\varepsilon\,d\xi = \int_0^L \Delta F(\xi,x)\,\mathrm{Sign}(x-\xi)\,d\xi \tag{4}$$

$$\Delta F(\xi,x) = 2\pi\int_0^\infty B_\varepsilon(\xi)\,\kappa_\varepsilon(\xi)\,E_2\left(\left|\tau_\varepsilon(\xi,x)\right|\right)d\varepsilon \tag{5}$$

Here B_ε is the equilibrium intensity, κ_ε the spectral absorption coefficient, E_2 the integral exponential functions. It is seen from (2) and (4) that the solution reduces to the space integration of the partial characteristics $\Delta I(\xi,x)$ and $\Delta F(\xi,x)$ that can be calculated according to (3), (5). These partial characteristics are determined by the distribution of spectral emissivity $j_\varepsilon(\xi) = B_\varepsilon(\xi)\,\kappa_\varepsilon(\xi)$ along the ray (this relation is local) and optical distance $\tau_\varepsilon(\xi,x) = \int_\xi^x \kappa_\varepsilon(y)dy$ τ_ε between the points of source and sink. The dependence on the spatial distribution of the thermodynamic variables $\tau_\varepsilon(\xi,x)$ has an integral character along the ray length. It is insensitive to the details of the profile of T and ρ (T, P) between points ξ and x. Therefore there is a possibility to integrate over quantum energy using the approximation of spatial profiles $T(y), \rho(y)$. For this purpose works [4, 5] have used linear continuous splines. Approximation with the help of the linear splines well describes radiation heat transfer in a volume with small differences of parameters [14]. We have generalized this technique, having included discontinuous splines into consideration [15–16].

It is seen that partial characteristics (3), (5) can be calculated if the temperature and density (pressure) profile between the points ξ and x is specified and the spectral coefficient of absorption $\kappa_\varepsilon(T,\rho)$ is known. In the simplest case the splines are linear. Then linear relations describe space distribution of plasma parameters. The approximation by the linear splines yields:

$$T(z) = T'_x + (T_\xi - T'_x)z, \qquad \rho(z) = \rho'_x + (\rho_\xi - \rho'_x)z, \qquad z = y/|\xi - x|, \qquad 0 \le z \le 1 \tag{6}$$

The quantities T'_x and ρ'_x can be taken or their values on true structure (T_x, ρ_x), or the values when linear splines can correspond to the real profile [5, 14]:

$$\int_\xi^x T(y)dy = \frac{1}{2}(T_\xi + T'_x)(x-\xi), \qquad \int_\xi^x \rho(y)dy = \frac{1}{2}(\rho_\xi + \rho'_x)(x-\xi). \tag{7}$$

In this case $\Delta I(\xi,x) = \Delta I(T_\xi, T_x, \rho_\xi, \rho_x, |x-\xi|)$, i.e. the tables of partial characteristics are five-dimensional. Linear splines provide the asymptotically correct behavior of the solution of the transfer

equation in the limits of small and large optical thickness. For a transparent medium, where $\tau_\varepsilon << 1$ in the whole range of the spectrum, the result is correct due to the absence of attenuation, and the distribution of radiation sources is always calculated on the true profile of the parameters T and ρ In the opposite case $\tau_\varepsilon >> 1$, the intensity of the radiation $I(x)$ (or the flux) is determined by a small neighborhood of the point x, and here the linear approximation is usually quite sufficient.
Applying the discontinuous approximation, we obtain:

$$T(z) = \begin{cases} T_\xi, & 0 \le z < \alpha_T \\ T_x, & \alpha_T \le z < 1 \end{cases}, \qquad \rho(z) = \begin{cases} \rho_\xi, & 0 \le z < \alpha_\rho \\ \rho_x, & \alpha_\rho \le z < 1 \end{cases}. \tag{8}$$

Parameters α_T and α_ρ are determined from the profiles of T and ρ:

$$\int_\xi^x T(y)\,dy = (x-\xi)\left[\alpha_T T_\xi + (1-\alpha_T)T_x\right], \quad \int_\xi^x \rho(y)\,dy = (x-\xi)\left[\alpha_\rho \rho_\xi + (1-\alpha_\rho)\rho_x\right] \tag{9}$$

It is clear, that if points ξ and x are on the different sides of physical discontinuity and the parameters on the left and on the right of it are homogeneous, we have a precise solution of the radiation transfer equation. In the latter case, partial characteristics are the functions of seven variables $\Delta I(T_\xi, \rho_\xi, T_x, \rho_x, |\xi - x|, \alpha_T, \alpha_\rho)$.

4. The tables of partial characteristics

In composing tables of partial characteristics one should calculate the optical thickness $\int_\xi^x \kappa_\varepsilon(y)dy$ between the point of the source and the point of observation accurately. For this it was assumed that on the portion of the path lying within the limits of the four nodes of the table of optical properties $T_i\rho_j, T_i\rho_{j+1}, T_{i+1}\rho_j, T_{i+1}\rho_{j+1}$ the mass coefficient of absorption K_ε can be represented in the form

$$\ln K = \ln K_1 + (\ln K_2 - \ln K_1)\frac{z - z_1}{z_2 - z_1}, \tag{10}$$

where K_1 and K_2 are the corresponding coefficients at the points of intersection of the ray with the boundaries of the given rectangle, and $z_2 - z_1$ is the portion of the path $X = |x - \xi|$ in it. Then the optical path between the points ξ and x is

$$\int_\xi^x \kappa dy = (x-\xi)\sum_i K_i \frac{z_{i+1} - z_i}{\ln(K_{i+1}/K_i)}\left\{\frac{K_{i+1}}{K_i}\left[\rho_{i+1} - \frac{\rho_{i+1} - \rho_i}{\ln(K_{i+1}/K_i)}\right] - \left[\rho_i - \frac{\rho_{i+1} - \rho_i}{\ln(K_{i+1}/K_i)}\right]\right\} \tag{11}$$

The contribution of an element of the table with a constant mass coefficient of absorption to the optical path is $(x-\xi)K_i(z_{i+1} - z_i)(\rho_i + \rho_{i+1})/2$.

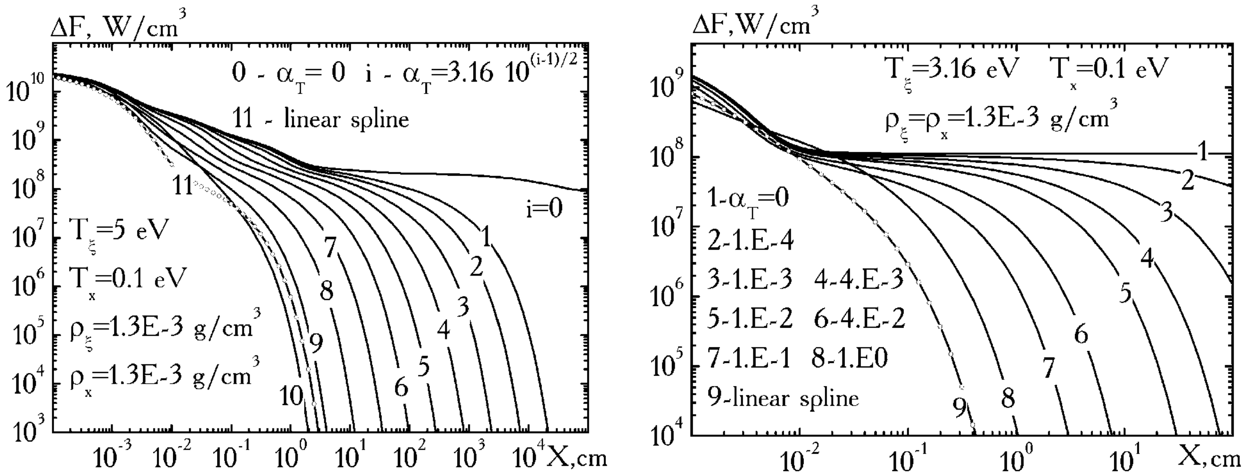

Fig. 5. Dependence of the air and aluminum partial fluxes on α_T and $X = |x - \xi|$.

The partial characteristics of air, carbon and aluminum have been calculated and tabulated in the approximation of linear and discontinuous splines. Some results for the partial fluxes are given in Fig. 5. Evidently that function ΔF is very sensitive to value α_T, what is connected with strong distinction in absorption coefficients at T_ξ and T_x. It is necessary to note the essential discrepancy in partial flux behavior at X increase. The linear splines give much stronger attenuation of ΔF in comparison with the discontinuous splines. In the latter case, depending on the location of points ξ and x in relation to the temperature front, the radiation passes a greater or a smaller part of the way against the low temperature background where due to a transparency window it is practically not absorbed. As a result the discontinuous description may provide an output of radiation from plasma volume that really takes place in a spectral range where absorption is small.

We note that radiation transfer under conditions where the spatial gradients of the pressure are small is much more convenient to calculate using temperature and pressure as independent parameters. In this case, due to the constancy of one of them ($P = \mathrm{const}$) the tables of partial characteristics turn to be more compact.

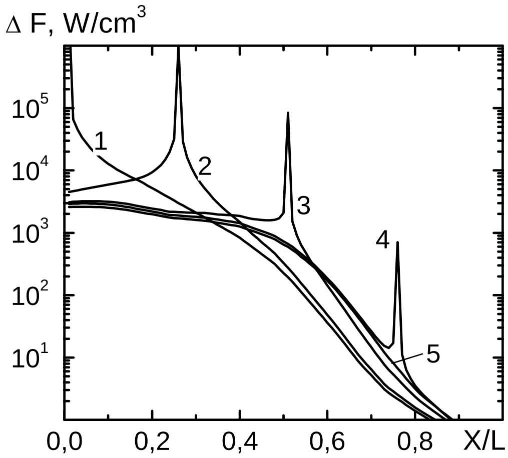

Fig. 6 Profiles of partial flux for the next points 1–X/L=0, 2 – 0.25, 3 – 0.5, 4 – 0.75 and 5 – 1

It is necessary to note, that the problem of spatial integration of partial characteristics for reception of a field of radiation (formulas (2), (4)) may appear enough difficult at increase of optical density. In particular, the partial the flux (intensity) may change in a vicinity of point x on some orders, and resulting value F appears a small difference of the single-direction fluxes big and close on size. Fig. 6 shows spatial distribution of partial flux for points of the plane air layer (width $L = 10^2$ cm) in which the density is constant and the temperature changes from $T_0 = 2$ eV up to $T_1 = 0.5$ eV under law $T(x) = T_0 /[1 + (T_0 / T_1 - 1) \cdot (x / L)^2]^{-1}$.

An analysis showed that in this case the most effective way to calculate the radiation is to use tables containing partial characteristics *x* integrated over a linear profile of the parameters (e.g., $G(T_\xi, T_x, \rho_\xi, \rho_x, X) = \int_\xi^x \Delta F(\xi, x) d\xi$) Clearly they can be used only in the local vicinity of the point x, where the linear approximation for the profile of T and ρ is justified.

5. Results and discussion

The verification of the method has been carried out by the comparison of the spectral calculation results and the data, obtained by the integral method. The first computation is made using approximately 1500 spectral groups. The basic results concerning use continuous linear splines, are given in [14].

As an example of application of continuous and discontinuous splines, we consider the propagation of a plane radiate wave from the zone of instantaneous energy-release. The wave moves in the plasma the density of which is assumed to be constant ($\rho = 1.29 \cdot 10^{-3}$ g/cm^3). The area of the energy-release is placed in a symmetry plane. An initial temperature is the following

$$T(x) = T_0 \left[1 + A(|x| / L)^\sigma\right]^{-1} \quad |x| \le L, \qquad T(x) = T_1 \quad |x| > L$$

Here $T_0 = 3$ eV – the initial temperature at $x = 0$, $L = 0.5$ cm – the half-width of a hot area, $\sigma = 10$ – the parameter of initial inhomogeneity, $A = T_0 / T_1 - 1$, $T_1 = 0.1$ eV – the background temperature. Since the only mechanism of energy transfer, which is taken into account, is heat radiation, the equation of energy can be represented as

$$\frac{\partial E}{\partial t} = -\frac{1}{\rho}\frac{\partial F}{\partial x}, \qquad E = C_v T, \qquad F = \int_0^\infty F_\varepsilon d\varepsilon$$

The tabular values of specific heat C_V have been utilized. Further we shall consider the results of the calculations. The space-time distributions of temperature and radiation flux are shown in Fig. 7 for

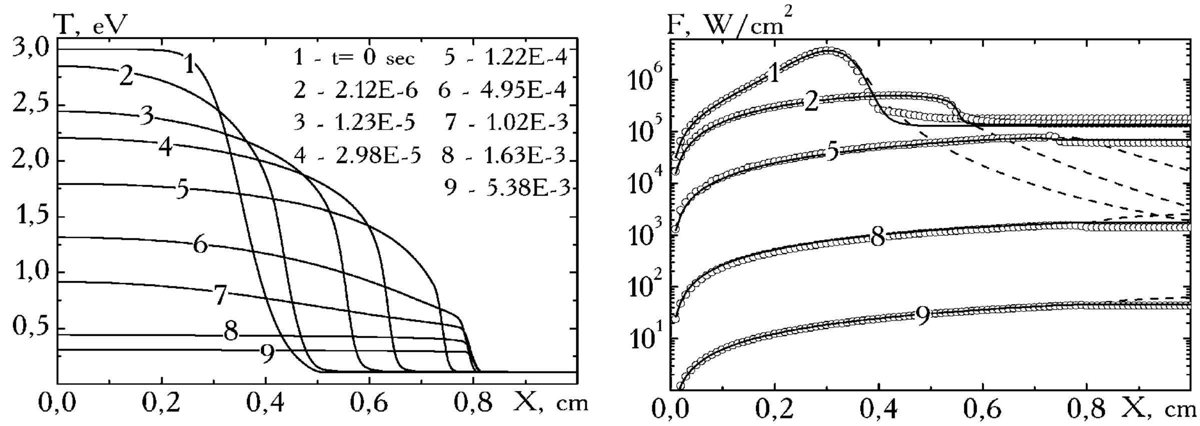

Fig. 7. Temperature and radiant flux profiles in sequential time instants. Aluminum plasma.

Al-plasma. The optical thickness of the layer strongly changes depending on quantum energy and T. The radiation with $\varepsilon > 5.97$ eV has large opacity in cold plasma. The absorption of this radiation in the vicinity of the wave front causes its motion. Quanta with smaller energy (except for spectral lines) escape from the plasma volume without absorption causing its radiative cooling. With cooling plasma the spectrum of its radiation moves to the area of transparency. At late stages the radiation emission has a volumetric character. In order to compare the spectral and the integral calculations, the space distribution of the heat flux at different instants is illustrated in Fig.7. The solid curves show the results of the calculations, which use spectral approximation. The dashed curves have been obtained with the use of the MPC (linear splines). The curves marked with circles have also been obtained by this method where both the linear and the discontinuous approximation of space variables is employed depending on the behavior of temperature between points ξ and x. It is obvious, that in the latter case there is quite satisfactory correspondence between the spectral and the integral description of the radiation transfer.

Fig. 8 shows the time dependence of the radiant flux from the plasma surface. At its change by over 7 orders of magnitude, the distinctions in results are less than 1.5 times. Thus the employment of the method of partial characteristics with the combined approximation of the plasma parameter pro-files enables the satisfactory determination of the exiting radiation.

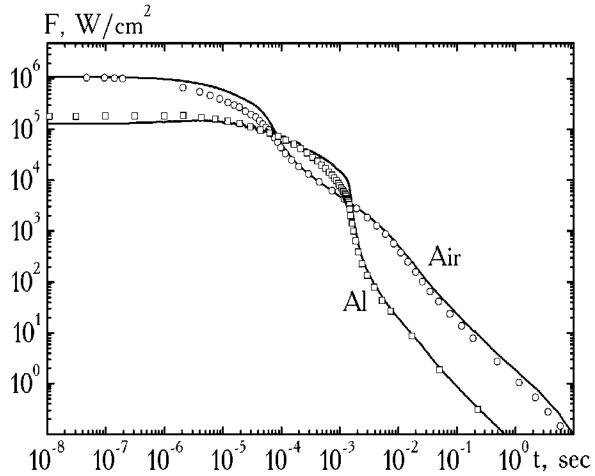

Fig. 8. Time dependence of the radiant flux from the plasma surface. Solid curves – the spectral calculation, circular curves – the calculation using modified the MPC.

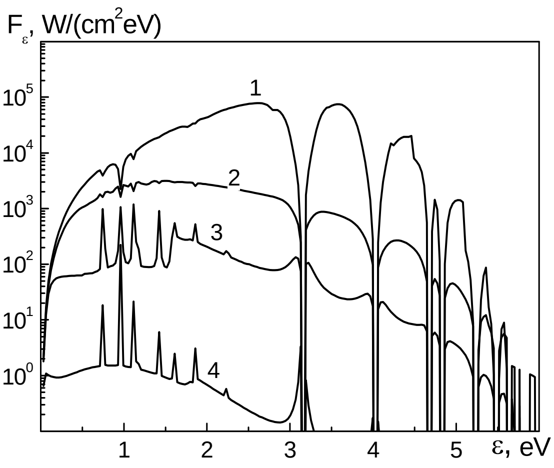

Fig. 9 Spectral radiation flux from the aluminum plasma: 1 – t = 2 μs , 2 – 2 ms, 3 – 3, 4 – 9 ms.

Representation about complicated character of radiation transfer gives fig. 9, where the exit spectral radiant flux is given for different point of time.

6. Conclusion

The data presented show that in describing radiation transfer in the case where the actual spectrum is characterized by many hundreds of intense lines and recombination continua where the intensity changes by an order of magnitude, use of the method of partial characteristics with certain modifications provides rather high accuracy of the determination of integral fluxes and intensities of radiation. This gives hope that this method will be used successfully for solving problems of non-stationary radiation gas dynamics.

Acknowledgements

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