ON THE ACOUSTICS OF A HIGH-SPEED ROTATING COMPRESSIBLE FLUID

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Influence of rotation on wave processes was considered in the classical works related to the theory of the wave phenomena at ocean and atmosphere, however for the acoustic waves in the compressible medium to the account of this influence until recently was not given due attention. During the present period interest to this problem continuously grows. For example, in works [1,2] necessity of the account of influence of rotation on strong acoustic fields for a wide class of technical devices is underlined and a number of vital issues of acoustics of a rotating gas is considered. However, in [1,2] are used barotropic models, inapplicable to acoustic processes in the nonhomogeneous (stratified) medium. Really, in spite of the fact that in nondissipative processes entropy s of each element of medium remains to a constant, for the stratified medium at various elements of medium entropy is various, and, hence, process will not be isentropic [3]. As a result, even at absence of dissipation in the acoustic process occurring in the stratified medium, pressure p is not function only of density ρ – use of two-parametrical model (for example, use the equation of a state of a kind $p = p(\rho, s)$) becomes necessary [4,5]. Thus, both the basic equations and concrete results of acoustics of high-speed rotating medium, received within the limits of barotropic models, demand corresponding correction [6].

Let's consider the cylinder rotating together with a fluid around of a vertical axis with constant angular speed $\vec{\Omega}$. In a system of reference rotating together with them, upon an element of fluid act the sum of gravitational, centrifugal and the Coriolis forces:

$$\vec{G} = -\overrightarrow{grad}\,(\Phi_g(z) - \tfrac{1}{2}\Omega^2 r^2) + 2\left[\vec{V}\times\vec{\Omega}\right].$$

From the system of hydrodynamic equations in linear approximation is deduced the closet vector equation which completely describes the dynamics of the nondissipative wave processes occurring on a background of any mechanically-equilibrium state in an uniformly rotating compressible liquid (gas) with any two-parametrical equation of state:

$$\ddot{\vec{V}} - \overrightarrow{grad}\left(c_0^2\, divV\right) = \overrightarrow{grad}\left(\vec{V}\cdot\vec{G}\right) + \left(\gamma_0^* - 1\right)\vec{G}\, divV + 2\left[\dot{\vec{V}}\times\vec{\Omega}\right], \qquad (1)$$

where $\gamma_0^* = c_0^2 \dfrac{d\rho_0}{dp_0}$. All variety of thermodynamic properties of such fluid and versions of background states (including thermally nonequilibrium state) enter into (1) only through two functions: $c_0^2(p_0, \rho_0)$ and $\gamma_0^*(p_0, \rho_0)$. For spatially-adiabatic $(\overrightarrow{grad}\, s_o = 0)$ background state the parameter $\gamma_0^* = 1$, but for isothermal – coincides with $\gamma_0 = \gamma(p_0, \rho_0)$, where $\gamma = C_P / C_V$. Thus, the member $\left(\gamma_0^* - 1\right)\vec{G}\ divV$ (which would be absent at use of the barotropic models) can have the same order, as members considered on this model.

Exact solution of (1) for generalized radial modes of a rotating cylindrical resonator containing an ideal gas is found. On the base of this solution are obtained: universal dependence of resonant frequencies on angular velocity and radius and analytical expressions for distributions of amplitudes of acoustic pressure and speed.

References

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