TRANSLATIONAL RELAXATION OF THE RAYLEIGH AND LORENTZ GASES IN THE FRONT OF SHOCK WAVES

O. V. Skrebkov, A. L. Smirnov, and A. N. Dremin

Institute of Problem of Chemical Physics of Russian Academy of Sciences,Chernogolovka, 142432, Russia
e-mail: skreb@icp.ac.ru

The present study is devoted to an investigation of translational relaxation immediately behind the shock wave front in a gas by the kinetic description, i.e., by the formulation and solution of kinetic equations. In particular, the latter is necessary for testing (in the simplest limit cases) the numerical simulation methods which (in the case of gas phase) remained laborious and not satisfactory on precision.

The principal moments of our kinetic problem formulation are based on the approaches elaborated in the classical works [1-5] and taking into consideration such a circumstance that the problem of translational relaxation in the shock wave is non-isotropic one in momentum and/or energy space (the direction of shock wave propagation is distinguished). These moments consist in the following. The Rayleigh gas is a spatially isotropic two-species gas with a very dilute subsystem of heavy mass points dispersed in a heat bath of light particles. The Lorentz gas is the "inverse" of the Rayleigh gas in that the dilute subsystem of light particles is dispersed in a heat bath of heavy particles. For this cases, it is possible to carry through the binary collision dynamics exactly in deriving the linearized Boltzmann equation which describes the relaxation of this systems. This in turn permits a very detailed analysis of the passage from the integral Boltzmann equation (the master equation) to the differential Fokker-Planck equation. In the vector form, this equation is

$$\frac{Df(\vec{c},t)}{Dt} = div\ j\,, \qquad j = f^{(0)}(\vec{c})\mathbf{B}(\vec{c})\frac{\partial}{\partial\vec{c}}\left[\frac{f(\vec{c},t)}{f^{(0)}(\vec{c})}\right], \tag{1}$$

here $f(\vec{c},t)$ is the velocity distribution, $f^{(0)}(\vec{c}) = f(\vec{c},\infty)$ is the equilibrium velocity distribution.

Herein, the precise analytical expression for the diffusion tensor coefficient, $\mathrm{B}(\vec{c})$, in the general case on the mass ratio of admixture and heat bath particles has been obtained from the solution of the corresponding dynamic collision problem. In the course of the study, Eq. (1) was considered in the two following cases: (i) the case of spherical symmetry when a gas as a whole is at rest (the velocity space is uniform and isotropic; 1D problem), (ii) the case of cylindrical symmetry when a gas as a whole has a flow velocity component in the direction of shock wave propagation (2D problem).

1D PROBLEM (the case of spherical symmetry). In the spherical coordinates, after averaging over the angles and the passage to energies, the diffusion coefficient, $B(x,\varepsilon)$, becomes a scalar function of dimensionless energy, $x=E/kT$, and parameter, $\varepsilon = m/M$; M is an admixture molecule mass, m is the heat bath molecule mass, T is the heat bath temperature, $E=Mc^2/2$. $B(x,\varepsilon)$ increases with increasing x. In the first case (Rayleigh gas) for the practically interesting energy range, $x \le 30$, $B(x) \approx 4\cdot\sqrt{\varepsilon} = const$. For this limit ($M >> m$), from our general formulas, we have the result formerly obtained in works [3,4].

2D PROBLEM (the case of cylindrical symmetry). The problem of translational relaxation in the shock wave is the two-dimensional one into the coordinates c_z and c_r (z is the coordinate in the direction of shock wave propagation). After passage in the vector Eq. (1) to the cylindrical coordinates and averaging over the angle, as a result of corresponding calculations, we come to the formulation of the 2D problem in the coordinates x_r, x_z. Here, $x_r=E_r/kT$, $E_r=Mc_r^2/2$; $x_z=E_z/kT$, $E_z=Mc_z^2/2$. In the case of the 2D problem, the character of the dependences $B_r(x_r, x_z)$ and $B_z(x_r, x_z)$ is retained (B_r and B_z increase with increasing x_r and x_z), i.e., the relaxation process at higher energies proceeds faster. The latter means that the super-equilibrium population of the "high-energetic tails of the distribution function" can decrease only during relaxation process.

As an initial condition, the Maxwellian distribution with the temperature of admixture molecules, T_0, (the gas temperature before the shock wave) and the flow velocity component immediately behind the shock wave, u, are set. In the coordinate system connected with a flow behind the shock wave, the heat bath gas is at rest and in equilibrium at the temperature T. The average initial velocity of admixture particles ("displacement" of the initial Maxwell distribution) is equal to u, and their distribution form corresponds to the temperature T_0.

For the case of the Rayleigh gas, the analytical solution in terms of the generalized Laguerre polynomials has been obtained; the 1D limit of this solution coincides with the result obtained formerly in works [3,4]. The translational relaxation in the Rayleigh gas has a 2D character. In the case of the Lorentz gas, the 1D description of the process is sufficient with a good precision. This circumstance can be used in the future while generalizing the problem for vibrating and dissociating molecules when the problem dimension will be increased.

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