GENERALIZED METHOD OF THE PARTIAL CHARACTERISTICS FOR DESCRIPTION OF RADIATION TRANSFER IN HIGH-TEMPERATURE GAS DYNAMICS

K. L. Stepanov, Yu. A. Stankevich, L. K. Stanchits and S. M. Zen'kevich

Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15, P.Brovka Street, 220072, Minsk, Belarus, e-mail: kls@hmti.ac.by

Including radiation greatly complicates solution of problems of high-temperature gas dynamics [1, 2]. A radiant flux and its divergence are spectrum-integral and angular-integral quantities that depend on the distribution of the parameters (temperature, density) in the entire volume under consideration. They should be determined rapidly and with sufficient accuracy on each time layer. It is obvious that direct integration over the spectrum in solving a gas dynamics problem is not used due to its laboriousness. The multigroup approximation frequently used for simplification of calculations, where the actual spectrum is replaced by several tens of spectral groups within the limits of which the absorption coefficients are averaged by one or another method, though giving a qualitatively accurate result, may greatly differ quantitatively from an approximation allowing for the actual spectrum. In particular, in [3] it is shown that under conditions where the radiation heat transfer is close to the limit of radiation thermal conductivity, the flow calculated in the multigroup approximation is several times lower than the actual one.

We used the spectrum-integral method of partial characteristics, which was developed in [4, 5], to describe radiation transfer in an actual spectrum. This method is developed for description of nonlinear wave motion in plasma. The method is based on representation of the radiant flux (for a plane layer) or the intensity in a prescribed direction in terms of spectrum-integral quantities. A databank of partial characteristics is composed for air, carbon, and aluminum. Detailed test calculations are made, and possible ways of accelerating calculations of a radiation field are studied.

The formal solution of radiation transfer equation on a ray of length L in the absence of sources on the boundaries is determined by the expression

$$I(x)=\int_0^\infty\int_0^L B_\varepsilon(\xi)\kappa_\varepsilon(\xi)\exp\left(-\left|\int_\xi^x \kappa_\varepsilon(y)dy\right|\right)\mathrm{Sign}(x\text{-}\xi)\,d\varepsilon\,d\xi=\int_0^L \Delta I(\xi,x)\,\mathrm{Sign}(x-\xi)\,d\xi\,. \qquad (1)$$

In the case of the plane layer this equation (1) is precisely integrated over the angular coordinate:

$$F(x)=2\pi\int_0^\infty\int_0^L B_\varepsilon(\xi)\kappa_\varepsilon(\xi)\,E_2\left(\left|\int_\xi^x \kappa_\varepsilon(y)dy\right|\right)\mathrm{Sign}(x\text{-}\xi)\,d\varepsilon\,d\xi=\int_0^L \Delta F(\xi,x)\,\mathrm{Sign}(x-\xi)\,d\xi\,. \qquad (2)$$

In (1), (2) B_ε is the equilibrium intensity, κ_ε the spectral absorption coefficient, E_2 the integral exponential functions. It is seen from (1), (2) that the solution reduces to the space integration of the partial characteristics $\Delta I(\xi,x)$ and $\Delta F(\xi,x)$

$$\Delta I(\xi,x)=\int_0^\infty B_\varepsilon(\xi)\kappa_\varepsilon(\xi)\exp\left[-\left|\tau_\varepsilon(\xi,x)\right|\right]d\varepsilon\,,\qquad \Delta F(\xi,x)=2\pi\int_0^\infty B_\varepsilon(\xi)\,\kappa_\varepsilon(\xi)\,E_2\left(\left|\tau_\varepsilon(\xi,x)\right|\right)d\varepsilon\,. \qquad (3)$$

There is a possibility to integrate over quantum energy using the approximation of spatial profiles $T(y), \rho(y)$. For this purpose works [4, 5] have used linear continuous splines. Approximation with the help of the linear splines well describes radiation heat transfer in a volume with small differences of parameters [6]. We have generalized this technique, having included discontinuous splines into consideration [7–8]. The approximation by the linear splines yields:

$$T(z)=T'_x+(T_\xi-T'_x)z,\qquad \rho(z)=\rho'_x+(\rho_\xi-\rho'_x)z,\qquad z=y/\left|\xi-x\right|,\qquad 0\le z\le 1\,. \qquad (4)$$

In this case the tables of partial characteristics are five-dimensional. Linear splines provide the asymptotically correct behavior of the solution of the transfer equation in the limits of small and large optical thickness. Applying the discontinuous approximation, we obtain:

$$T(z) = \begin{cases} T_\xi, & 0 \le z < \alpha_T \\ T_x, & \alpha_T \le z < 1 \end{cases}, \qquad \rho(z) = \begin{cases} \rho_\xi, & 0 \le z < \alpha_\rho \\ \rho_x, & \alpha_\rho \le z < 1 \end{cases}. \tag{5}$$

It is clear, that if points ξ and x are on the different sides of physical discontinuity and the parameters on the left and on the right of it are homogeneous, we have a precise solution of the radiation transfer equation.

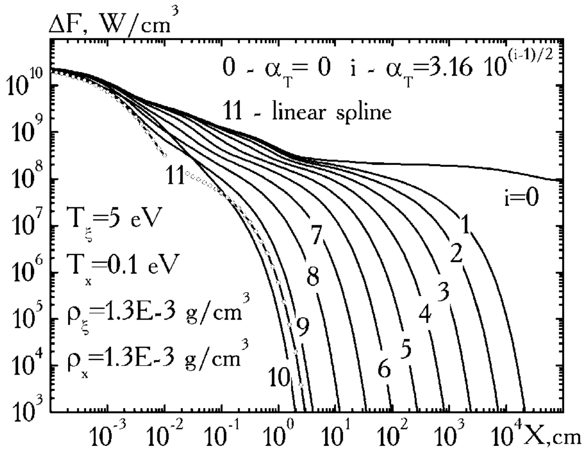

Fig. 1. Partial flux as function of α_T and $|x-\xi|$ for air

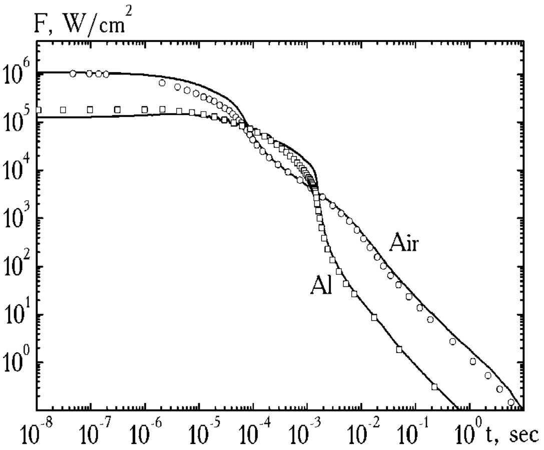

Fig. 2. Radiation flux from the plasma surface

Fig. 1 presents some results for the partial fluxes in the aluminum plasma. It is seen that ΔF is very sensitive to value α_T, what is connected with strong distinction in absorption coefficients at different T. As an example of application of continuous and discontinuous splines, it is considered the propagation of a plane radiate wave from the zone of instantaneous energy-release. The wave moves in the plasma the density of which is assumed to be constant. Fig. 2 shows the time dependence of the radiant flux from the plasma surface. At its change by over 7 orders of magnitude, the distinctions in results of spectral calculation (solid curves) and the calculation using modified the MPC (circular curves) –are less than 1.5 times. Thus the employment of the method of partial characteristics with the combined approximation of the plasma parameter profiles enables the satisfactory determination of the exiting radiation.

The data presented show that in describing radiation transfer in the case where the actual spectrum is characterized by many hundreds of intense lines and recombination continua where the intensity changes by an order of magnitude, use of the method of partial characteristics with certain modifications provides rather high accuracy of the determination of integral fluxes and intensities of radiation.

References

[1] Mihalas D.M., Mihalas B.W. (1984) Foundations of radiation gas dynamics, N. Y.

[2] Chetverushkin B.N. (1985) Mathematical simulation of problems of the dynamics of a radiating gas, Nauka, Moscow.

[3] Stepanov K.L., Stanchits L.K., Stankevich Yu.A. (2000) Radiation of Strong Shock Waves in Air: Part I, II. High Temperature, Vol. 38, No 2, pp. 182-190, No 3, pp. 367-373.

[4] Sevast'yanenko V.G. (1980) Radiation heat transfer in a real spectrum, Doctor's Thesis on Physical and Mathematical Sciences, Novosibirsk.

[5] Soloukhin R.I. (ed.) (1984) Radiation heat transfer in high-temperature gases: Reference Book, Energoatonizdat, Moscow.

[6] Stepanov K.L., Panastnko L.N., Romanov G.S. el al. (1999) Integral transfer of radiation of the actual spectrum in problems of radiative plasma dynamics. JEPTER, Vol. 72, No 6, pp. 1065-1074.

[7] Zen'kevich S.M., Stanchits L.K., Stepanov K.L. (2003) Radiation transfer in the presence of nonlinear waves in plasma. Proceeding of int. symposium "Radiative plasma dynamics-VI", pp. 7–12.

[8] Stepanov K.L., Stankevich Yu.A., Stanchits L.K. (2003) Radiation transfer in the problem of propagation of shocks, thermal and radiative waves. The integrated approach for actual spectrum. Int. J. High Temperature Material Processes, Vol.7, Issue 2, pp.169–175.