EVOLUTION OF TRANSIENT WAVE DESCRIBED BY HYPERBOLIC EQUATIONS

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The fluid flow governing equations are based on the conservation laws of: mass, momentum and energy. The system of conservation equations contains unknown fluxes. The fluxes could be coupled with the basic unknowns by means of phenomenological equations well known in the form of Newton, Fourier and Stokes laws. Such a system of equations (conservation and phenomenological ones) are of the parabolic type and corresponds to the situation, where signals propagate infinitely fast. The mathematical feature of such a system causes that any change of any parameter, for example temperature, in one place is observed immediately in a place far from the location of the initial change. This non-physical feature of the equations can be improved, assuming that the heat and stress fluxes are independent variables described by non-stationary evolution equations. The approach which enlarges number of independent variables is a base of the Extended Irreversible Thermodynamics – EIT [1]. It assumes that the properties of fluid depend not only on classical variables, but also on dissipative heat and viscous pressure fluxes.

These fluxes are described by transport equations, which cause the change of the type of the equations from parabolic to hyperbolic one. This approach eliminates the defect of the classical fluid mechanics equations, consisted on infinitely fast signal velocity. In the EIT approach the fluid flow is described by: mass conservation equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{\upsilon}) = 0,$$

momentum conservation equation

$$\frac{\partial \rho \vec{\upsilon}}{\partial t} + \nabla \cdot \left(\rho u \vec{\upsilon} + p\vec{I} + p^{\nu}\vec{I} + \vec{P}^{\nu} \right) = \rho \vec{f},$$

and energy conservation equation

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \vec{\upsilon} + \vec{q}) + \left(p\vec{I} + p^{\nu}\vec{I} + \vec{P}^{\nu} \right) : \nabla \vec{\upsilon} = 0,$$

where the latter variables mean: ρ – fluid density, $\vec{\upsilon}$ – velocity vector, p – thermodynamic pressure, p^{ν}– scalar bulk viscous pressure, $\vec{P}^{\nu}$ – tensor viscous pressure, $\vec{f}$ – vector of mass force, u – energy, $\vec{q}$ – heat flux, as well as the fluxes evolution equations. Fluxes evolution equations (Maxwell-Cattaneo) for heat, bulk pressure and viscous pressure have the following form:

$$\left(\frac{\partial \vec{q}}{\partial t} + \vec{\upsilon} \nabla \vec{q} \right) \theta_q = -\vec{q} - \lambda \nabla T$$

$$\left(\frac{\partial p^{\nu}}{\partial t} + \vec{\upsilon} \nabla \cdot p^{\nu} \right) \theta_{\nu} = -p^{\nu} - \xi \nabla \vec{\upsilon}$$

$$\left(\frac{\partial \vec{P}^{\nu}}{\partial t} + \vec{\upsilon} \nabla \cdot \vec{P}^{\nu} \right) \theta_p = -\vec{P}^{\nu} - 2\eta \vec{V}$$

where the latter symbols mean: θ_q – relaxation time of heat flux, θ_ν – relaxation time of bulk viscous pressure, θ_p – relaxation time of viscous pressure, λ – heat transfer coefficient, ξ – bulk viscosity, η – viscosity, V – deviatoric traceless tensor. Because of difficulties in solving such a system of equations the simplified model which takes into account effects of convection, diffusion as well as relaxation is analyzed. Limiting the fluid flow description to the one-dimensional, assuming that viscosity pressure tensor $\vec{\vec{P}}^{\nu}$, heat flux $\vec{q}$ are zero, and thermodynamic pressure p and density ρ is almost constant, the set of latter equations reduces to the following two equations only:

$$\frac{\partial \upsilon}{\partial t}+\upsilon\frac{\partial \upsilon}{\partial x}+\frac{1}{\rho}\frac{\partial p^{\nu}}{\partial x}=0, \qquad (1)$$

$$\left(\frac{\partial p^{\nu}}{\partial t}+\upsilon\frac{\partial p^{\nu}}{\partial x}\right)\theta_{\nu}=-p^{\nu}-\xi\frac{\partial \upsilon}{\partial x}\quad , \qquad (2)$$

where x is a spatial coordinate. These two equations describe 1D transport and interaction between velocity and bulk viscous pressure. In the vicinity of equilibrium (thermodynamic) pressure p, bulk viscosity ξ could be extended as follows:

$$\xi(p+p^{\nu})=\xi_0(p)+\frac{d\xi}{dp}p^{\nu} \quad .$$

Small disturbances analysis of one-dimension momentum and bulk viscous pressure evolution equations show that signal velocity a for short waves is limited and is described by the formula:

$$a=\upsilon\pm\sqrt{\left(\xi_0+\frac{d\xi}{dp}p^{\nu}\right)\frac{1}{\theta_{\nu}}} \quad . \qquad (3)$$

Bulk viscosity decreases in terms of density [2], so the same could be said about bulk viscosity dependence on pressure $d\xi/dp<0$.

The numerical solution of initial-boundary value problem for the equations (1), (2) is presented in Figure 1. Initial velocity and bulk viscous pressure are equal 0. On the left boundary, velocity increases linearly from 0 to 10 m/s in time 0.1s. In the test presented here it was assumed that relaxation time θ_{ν} =10^{-4}s, bulk viscosity ζ_0 =0.01m^2/s, ρ=1 kg/m^3 and $d\xi/dp$ =-0.01 m^{3}s/kg.

a) b)

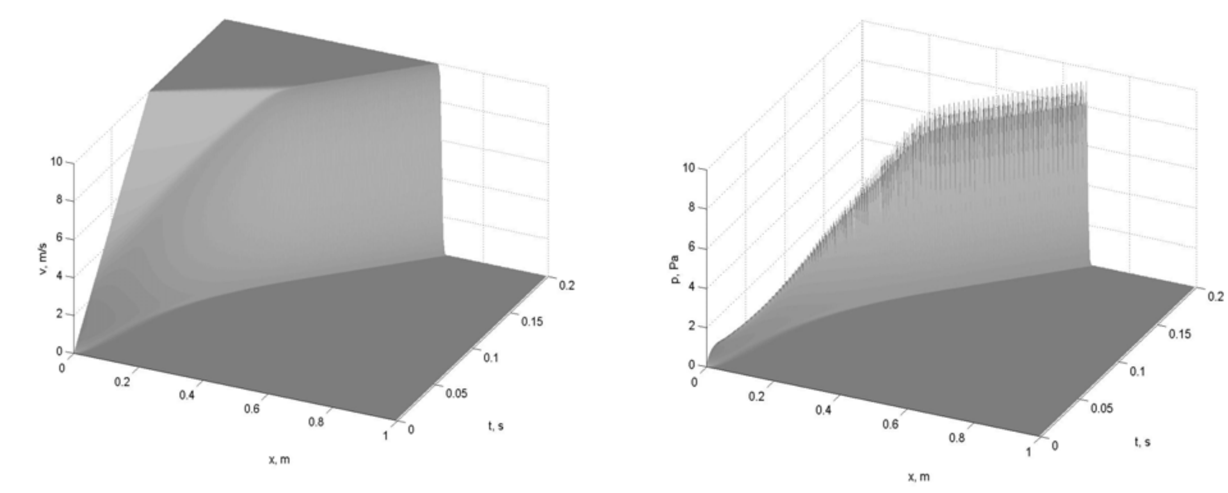

Fig. 1. a) Velocity in terms of time and space. b) Bulk viscous pressure surface in terms of time and space.

At the beginning, numerical solution of the problem shows that increase of velocity on the left boundary generates pressure pulse. Velocity wave accelerates because of high velocity amplitude on the left boundary, what in turns causes pressure amplification. Afterwards (approx. t=0.07s) higher pressure makes signal velocity lower -eq.(3), so finally wave velocity is constant and wave thickness is limited.

References

[1] Jou D., Casas-Vazquez J., Lebon G. Extended Irreversible Thermodynamic, Springer, 1998.

[2] Fernandez G.A., Vrabec J., Hasse H. A molecular simulation study of shear and bulk viscosity and thermal conductivity of simple real fluids, Fluid Phase Equilibria, 221 (2004), pp 157-163.