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THEORETICAL AND NUMERICAL STUDY ON THERMAL PROPERTIES OF FIBROUS INSULATION MATERIALS

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Abstract: In this work, the heat transfer coupling the heat conduction with the thermal radiation for two types of porous fibrous structures, including the randomly distributed fibrous media (RDFM) and directionally distributed fibrous media (DDFM), is theoretically and numerically investigated. The theoretical models and the Rossland approximation are respectively used to study the conductive thermal conductivity (CTC) and the radiative thermal conductivity (RTC) that implement with the aid of the Finite Volume Method (FVM) and Discrete Ordinate Method (DOM), respectively. The modeling results reveal that, for CTC of fibrous materials in the case of RDFM, the Random model and Hamilton model show the good agreements with the numerical method. Whereas, in the case of DDFM, the preceding models cannot predict the conductive thermal conductivity precisely. To overcome this issue, we propose a new model that take the fiber orientation, fiber length and thermal conductivity ratio of fiber and matrix phase into account. The proposed model enables to predict the conductive thermal conductivity precisely in DDFM. In addition, it is found that the CTC decreases exponentially with angle between the heat flux direction and fiber orientation. Since CTC is determined, the combined conductive and radiative thermal conductivity (CRTC) is calculated by theoretical model added with Rossland approximation as well as numerical method that is FVM combined with DOM. The results show that besides cases where material's optical thick is very small, the CRTC calculated by two methods show good agreement with each other, indicating the validity that the two methods used in predicting the CRTC of fibrous media.

Key words: fibrous media, thermal conductivity, combined conductive and radiative heat transfer

1. Introduction

Porous fibrous materials with the excellent thermal properties have attracted ever-increasing attention for civil and military applications, especially in the area of heat transfer as thermal insulation materials. Regarding the heat transfer in porous fibrous materials, there exist three transfer modes, including conduction, convection and radiation. The convection, however, can be ignored due to the small pore size in the porous fibrous insulation materials. Therefore, the conduction and radiation become the predominant heat transfer modes that need to be detailedly studied.

In studying the porous fibrous materials, past efforts have mainly focused on establishing models to achieve the effective thermal conductivity. Maxwell [1] firstly studied the effective thermal conductivity of composites with irregularly dispersed fillers in a continuous matrix. Using potential theory, the effective thermal conductivity of a system with spherical, non-interacting fillers in a continuous matrix was obtained by solving the Laplace equation. In order to take the effect of the shape of particles into account, Maxwell model was modified by Hamilton and Crosser [2]:

$$\lambda_{\mathrm{eff}} = \frac{\lambda_{\mathrm{f}} + (n-1)\lambda_{\mathrm{m}} - \phi(n-1)(\lambda_{\mathrm{m}} - \lambda_{\mathrm{f}})}{\lambda_{\mathrm{f}} + (n-1)\lambda_{\mathrm{m}} + \phi(\lambda_{\mathrm{m}} - \lambda_{\mathrm{f}})} \lambda_{\mathrm{m}}$$

where n is the empirical shape factor. In the case of the infinite cylinder, $n = 6$; while in the case of the spherical particle, $n = 3$, and Hamilton and Crosser model is in agreement with Maxwell model. Hasselman and Johnson [3] considered the effect of interfacial thermal resistance and derived the effective thermal conductivity of circular cylinder oriented perpendicularly to heat flow according to the Maxwell theory. However, this model is valid only for dilute fiber volume fraction for which interactions between the temperature fields of neighboring dispersions is negligible.

Among the above models, thermal conductivities of each phase and fiber volume fraction have been considered, whereas fiber orientation has not been taken into account. These types of models can be used to calculate the effective thermal conductivity of disordered fibrous material. Some other models have also been proposed for disordered fibrous medium. Zhang et al. [4] and Daryabeigi [5] utilized the parallel model to predict the effective thermal conductivity. Daryabeigi [6] also proposed a combined model as follows:

$$\lambda_{\mathrm{c}}(\theta) = \{\phi \cdot \lambda_{\mathrm{f}} + (1-\phi) \cdot \lambda_{\mathrm{gel}}\} A + \left(\frac{\lambda_{\mathrm{f}} \cdot \lambda_{\mathrm{gel}}}{\phi \cdot \lambda_{\mathrm{gel}} + (1-\phi) \cdot \lambda_{\mathrm{f}}}\right)(1-A)$$

where A and $1\text{-}A$ are the fractions of heat transfer in parallel and series mode, respectively. Wang and Pan [7] proposed several kinds of models for predicting the effective thermal conductivity of complex multiphase materials in their review article. Liu [8] systematically reviewed the models that are used for calculating effective thermal conductivity of porous media in his doctoral thesis. According to their descriptions, several models can be used to predict the effective thermal conductivity of fibrous media with disordered fibers, such as the Hamilton model and random model.

While for fibrous media with specific orientation, some effort has also been made to show the relationship between thermal properties and the fiber orientation. Jagjiwanram and Singh [9] proposed a combined model based on the Parallel model and Series model which take the angle contained by fiber axis direction and heat flux into account. Pitchumani and Yao [10] presented a unified treatment using the tool of local fractal dimensions to reduce the geometric complexity of the relative fiber arrangement in the composite. A generalized unit cell is constructed based on the fiber volume fraction and local fractal dimensions along directions parallel and transverse to the heat flow direction. The thermal model resulting from a simplified analysis of this unit cell is shown to be very effective in predicting the conductivities of composites with both ordered as well as disordered arrangement of fibers.

Zhou et al. [11] proposed a concept of heat transfer passages, and used it to evaluate the high filler leading composite materials in which the effects of filler orientation and distribution on the effective thermal conductivity can be considered. However, their model supposes that heat is transferred in parallel approximately in composite materials and filler volume fraction is high, which limits its use for effective thermal conductivity.

Fu and Mai [12] studied the thermal conductivity of short-fiber-reinforced polymer composites by taking the effect of fiber orientation distribution and length distribution into account. The results showed that the thermal conductivity increases with mean fiber length (or mean aspect ratio) but decreases with mean fiber orientation angle with respect to the measured direction. Zou et al. [13] analytically derived expression for transverse thermal conductivity of unidirectional fiber composites with thermal barrier based on the electrical analogy technique and on the cylindrical filament-square packing array unit cell model (C-S model). Their theoretical predictions with or without thermal barrier are found to be in excellent agreement with the existing analytical model and experimental data. Koráb et al. [14] experimentally measured thermal conductivity in two main orientations to the fiber direction by the laser-flash technique. Measurements revealed decreasing thermal conductivity as the fiber volume content in the composite increased due to the low thermal conductivity of fiber compared with the copper matrix, and the transverse thermal conductivity of the unidirectional samples presented much lower values in comparison to the longitudinal one. Wang et al. [15] used Lattice Boltzmann method to numerically investigate the ETC fibrous media. In their study, a random generation-growth method for generating micro morphology of natural fibrous materials is proposed and a highly efficient lattice Boltzmann algorithm for solving the energy transport equations is utilized. The simulation results indicate that the effective thermal conductivity of fibrous material increases with the fiber length and approach a stable value when the fiber tends to be infinite long.

The above articles considered only the heat conduction in fibrous media. In general, these previous studies mostly used some inexact models to predict the effective thermal conductivity of fibrous material [4-6, 9], or applied regular structure and unit cell model to study the effective thermal conductivity [10, 13], or focused on thermal conductivity either in direction that longitudinal or transverse by using single fiber model (unidirectional fiber composites) to analyze the heat transfer of fibrous media, few have thoroughly show the effect of fiber orientation as well as fiber length on thermal conductivity. It is also confusing that in such lots of models that provided in previous studies, which model should be used for for predicting effective thermal conductivity of fibrous media in practical application. So one aim of this study is to show how fiber orientation influences its effective thermal conductivity and to ascertain effective thermal conductivity prediction models for randomly distributed fibrous media (RDFM) and directionally distributed fibrous media (DDFM).

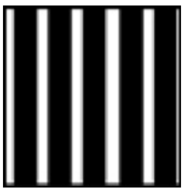
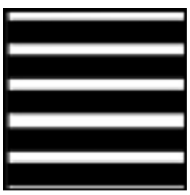
While fibrous media used as insulation material under high-temperature environment, radiative heat transfer becomes significant and should be investigated combined with heat conduction. Many researchers have studied combined radiative and conductive heat transfer in fibrous material experimentally and analytically. Tong et al. [16, 17] studied the radiative heat transfer in a grey medium made up by fibers distributed randomly in space by using the two-flux approximation model and obtained good agreement between the experimental and theoretical model results. Jeandel et al. [18] also used two-flux model to study the radiative heat transfer problems in glass fiber mediums. Lee [19, 20] considered the effect of fiber orientation on radiative heat transfer based on the classical Mie theory, and solved radiative properties of three pattern orientation fiber media. Zhang et al. [4] and Daryabeigi [5] investigated the high temperature characteristics of fibrous insulating materials numerically and experimentally. Baillis et al. [21, 22] used Rossland equation to account for the radiative heat transfer through foam porous material. Considering the effect of anisotropy, they used a size factor to modify the original radiative properties. Placido et al. [23] developed a geometrical cell model for applying to the prediction of radiative and conductive foam insulation properties. The gas and solid conductivities were determined by empirical model while the radiative conductivity was determined by Rossland equation. Zhao et al. [24] applied a similar method as Placido et al. [23] to study the heat transfer problems through fibrous insulating materials. In these studies, two flux approximation and Rossland approximation were used to account for the radiative heat transfer in fibrous media. Two flux approximation is the simplest multi-flux approximation which is derived under the assumptions that the energy transfer is one-dimensional and the intensity is isotropic for all radiation with components in the positive coordinate direction and in the negative direction with a different value. While Rossland approximation requires that the media is optically dense media and radiation travels only a short distance before being scattered or absorbed. For this situation, it is possible to transform the integral relation for the radiative energy into a diffusion relation like that for heat conduction.

Discrete ordinate method (DOM) is a commonly used method that can be applied to account for the radiative heat transfer [25] in insulation materials. It discretes the general radiation transfer relations into a set of equations for an intensity that is angularly averaged over each of a finite number of ordinate directions. Then these discretized radiation transfer equation are solved numerically. DOM was first proposed by Chandrasekhar [26], and then Lathrp [27] applied this method to solve the neutron transport problems. Truelove [28, 29] and Fiveland [30-32] did a lot of on the application of DOM in radiative heat transfer. Fiveland [30] and Kim and Baek [33] investigated the combined radiative and conductive heat transfer in two dimensional square rectangular enclosure by using DOM as well as finite volume method (FVM). Sakami et al. [34] extended the method into two dimensional complex geometry which also take combined conductive and radiative heat transfer into account. Liu and Tan [35] numerically analyzed the transient coupled radiative-conductive heat transfer process in a two-dimensional semitransparent cylinder caused by a pulse irradiation at one end of the cylinder. While Lacroix et al. [36] and David et al. [37] considered the coupled radiative-conductive heat transfer in non-grey medium. Due to the convenience of dealing with incoming scattering intensity and combining with other governing equation, DOM is considered to be a promising method in radiative heat transfer problems.

DOM has no limitations such as one-dimensional, isotropic as well as optically thick which two-flux approximation and Rossland approximation have, and it also works in low computational power as compared with other numerical methods such as Zone method and Monte Carlo simulation. Owing to the preceding mentioned merits, in this work, DOM is used to investigate the radiative heat transfer in fibrous media. At the same time, Rossland approximation is also used as comparison with DOM. Based on the DOM and Rossland approximation, the theoretically and numerically investigations on the coupled conductive and radiative heat transfer in fibrous media were conducted, and the overall effective thermal conductivity for both RDFM and DDFM was achieved.

2. Theoretical thermal conductivity models for fibrous media

Table 1 Parallel model and Series model of effective thermal conductivity

Parallel model		$\lambda_e = \phi \cdot \lambda_f + (1-\phi)\cdot \lambda_m$
Series model		$\lambda_e = \dfrac{\lambda_f \cdot \lambda_m}{\phi \cdot \lambda_m + (1-\phi)\cdot \lambda_f}$

In fibrous media, fibers always distribute randomly or distribute directionally. For directionally distributed fibers, the most two famous models are Parallel model and Series model. Assuming heat flow is in the vertical direction, fiber's and matrix's thermal conductivity are λ_f and λ_m, fiber's volume fraction is ϕ, then these two models can be expressed as Table 1 shows. Parallel model and Series model give the maximum and minimum limits of effective thermal conductivity for two-phase media.

While for fibrous media with other orientation, for example, Fig. 1 shows a structure in which the fiber orientation and heat flow direction contains an angle of θ, for this kind of fibrous media, few models has been proposed to account for the fiber orientation's impact on thermal conductivity.

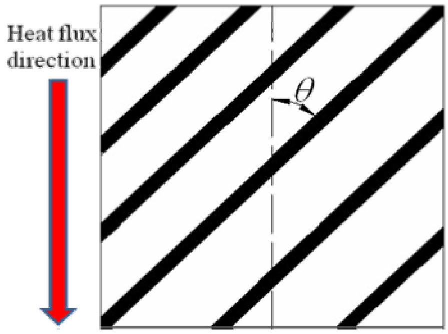


Fig. 1 Structure of fibrous media with a specified orientation

$$\lambda_c^2(\theta) = \{\phi \cdot \lambda_f + (1-\phi)\cdot \lambda_m\}^2 \cos^2\theta + (\frac{\lambda_f \cdot \lambda_m}{\phi\cdot\lambda_m + (1-\phi)\cdot\lambda_f})^2 \sin^2\theta \quad (1)$$

Jagjiwanram and Singh [9] assumed that continuous and dispersed phases were in the form of parallel slabs and make an angle θ with the direction of heat flux. They resolved effective thermal conductivity into two components, one parallel to the heat flux and the other perpendicular to it, and then they gave a model that combined the two basic models for effective thermal conductivity as Eq.(1) shows.

$$\lambda_c(\theta) = \{\phi \cdot \lambda_f + (1-\phi)\cdot \lambda_m\} \cos\theta + (\frac{\lambda_f \cdot \lambda_m}{\phi\cdot\lambda_m + (1-\phi)\cdot\lambda_f}) \sin\theta \quad (2)$$

Similar to Eq.(1), a model can also be developed as Eq.(2). In this study, Eq.(1) is called Model One, and Eq.(2) is called Model Two.

In most cases, fibers are distributed randomly in matrix material, which means that fibers had same weight in each direction. For this kind of RDFM, several models for two-phase effective thermal conductivity have been developed, such as Hamilton model and Random model. Hamilton model can be expressed as Eq.(3).

$$\frac{\lambda_e}{\lambda_{opa}} = \frac{\alpha + (n-1) + (n-1)(\alpha-1)\phi}{\alpha + (n-1) + (1-\alpha)\phi} \quad (3)$$

where $\alpha=\lambda_f/\lambda_m$ is the ratio of fiber thermal conductivity and matrix thermal conductivity; ϕ is the volume fraction of fibers; n is the empirical shape factor. In the case of the spherical particle, $n = 3$ and in the case of the infinite cylinder, $n = 6$.

Random model is written as

$$\lambda_e = \lambda_f^{\phi} \lambda_m^{1-\phi} \quad (4)$$

Effective thermal conductivity models for DDFM and RDFM will be used and compared with results that are numerically calculated. According to the results, some models will be modified and some models will be ascertained for both DDFM and RDFM.

In order to take radiative heat transfer into account, Rossland equation is used to calculate the radiative thermal conductivity (RTC) of fibrous media. Rossland equation is a simplified model of Radiative Transfer Equation (RTE). When material has a great extinction coefficient, it can be treated as optically thick, then RTE is simplified similar to Fourier law, which is called Rossland approximation. Using Rossland approximation, RTC can be easily calculated and fortunately most insulation materials meet the optical thick condition; so it is used extensively when calculating the RTC of composite insulation materials [21, 23, 38-40].

According to Rossland equation, RTC can be calculated by the following equation,

$$\lambda_{\mathrm{rad}} = \frac{16\sigma^2 n^2 T^3}{3\sigma_{\mathrm{eR}}} \tag{5}$$

where σ is the Boltzmann constant, n is the ambient medium's refractive index, T is the material's mean temperature, and σ_{eR} is Rossland extinction coefficient. σ_{eR} is related to the spectral extinction coefficient of the material, and their relationship is defined as

$$\frac{1}{\sigma_{\mathrm{eR}}} = \int_0^\infty \frac{1}{\sigma_{\mathrm{e}\lambda}} \frac{\partial e_{\mathrm{b}\lambda}}{\partial T} \mathrm{d}\lambda \Big/ \int_0^\infty \frac{\partial e_{\mathrm{b}\lambda}}{\partial T} \mathrm{d}\lambda = \int_0^\infty \frac{1}{\sigma_{\mathrm{e}\lambda}} \frac{\partial e_{\mathrm{b}\lambda}}{\partial e_{\mathrm{b}}} \mathrm{d}\lambda \tag{6}$$

$\sigma_{\mathrm{e}\lambda}$ is the spectral extinction coefficient of fibrous media, $e_{\mathrm{b}\lambda}$ and e_{b} are spectral hemispherical emissive power and hemispherical emissive power. $\sigma_{\mathrm{e}\lambda}$ should include both the matrix media and fibers' extinction coefficients, and can be calculated briefly by

$$\sigma_{\mathrm{e}\lambda} = \sigma_{\mathrm{e}\lambda,\mathrm{m}} + \sigma_{\mathrm{e}\lambda,\mathrm{f}} \tag{7}$$

In Eq.(7), $\sigma_{\mathrm{e}\lambda,\mathrm{m}}$ and $\sigma_{\mathrm{e}\lambda,\mathrm{f}}$ are the spectral extinction coefficient of matrix and fibers.

As Rossland approximation gives an expression of RTC and theoretical models stated above give conductive thermal conductivity (CTC), a simplified model for combined conductive and radiative thermal conductivity (CRTC) is considered to add the CTC and RTC directly. This approximation assumes that the interaction between the two modes transfer processes is very weak. For this combination, the energy transfer is similar to heat conduction with a thermal conductivity that depends on temperature.

$$\lambda_{\mathrm{total}} = \lambda_{\mathrm{c,gel+f}} + \lambda_{\mathrm{rad,gel+f}} = \lambda_{\mathrm{c,gel+f}} + \frac{16\sigma n^2 T^3}{3\sigma_{\mathrm{e}R}} \tag{8}$$

3. Governing equations and numerical methods

3.1 Structure

Besides theoretical study, numerical method will also be applied on the study of heat transfer performance of fibrous media, thus the computational structure of fibrous media should be firstly constructed. Wang et al. [15] used a random generation-growth method for generating micro morphology of natural fibrous materials based on existing statistical macroscopic geometrical characteristics. In this paper, we used similar method to generate the structure of fibrous media that meet several specific conditions including fiber volume fraction, fiber orientation and fiber length.

First, a 200*200 squared grid system is formulated in which a pixel represents a fiber point or a matrix medium point. In this study we use "1" represents the fiber and "0" represents the matrix.

Second, parameters of fiber such as its diameter (D), length (L), centric coordinate (x, y) and orientation (θ) should be specified. These parameters could all be determined by a Random Number Based Method (RNBM). For example, for a 2 dimensional problem, assuming that fibers are located in the matrix uniformly with angles between $\theta_{\min}$ and $\theta_{\max}$, where angle is the angle contained by fiber longitudinal direction and the heat flux direction, $\theta_{\min}$ and $\theta_{\max}$ are the minimum and maximum value that angle can be got respectively. The fiber orientation can be fixed as follows,

$$\theta = R_\theta(\theta_{\max} - \theta_{\min}) + \theta_{\min} \tag{9}$$

In Eq.(9), R_θ represents the orientation random number. Similar to fiber orientation determination, the fiber centric coordinate can also be determined randomly according to Eq.(10), (11) respectively.

$$x = R_x(x_{\max} - x_{\min}) + x_{\min} \tag{10}$$

$$y = R_y(y_{\max} - y_{\min}) + y_{\min} \tag{11}$$

For simplicity, fiber diameter and length are both constant in our study, which are 0.005 and 0.4 compared with the width 1.0 of the system respectively.

When a fiber's orientation and centric coordinate as well as its length and diameter are fixed, this fiber could be specified. In numerical method, this can be realized by looping all the pixels to judge whether d_{pL}, the distance between a point and the line that be specified by fiber's orientation and centric coordinate is lower than $D/2$. In order to guarantee the fiber length not exceed L, the distance of the point and the centroid of fiber d_{pC}, should also be calculated and compared to $L/2$.

$$y = kx + b \tag{12}$$

$$d_{\mathrm{pL}} = \frac{|y - kx - b|}{\sqrt{1+k^2}} \tag{13}$$

$$d_{\mathrm{pC}} = \sqrt{(x-x_0)^2 + (y-y_0)^2} \tag{14}$$

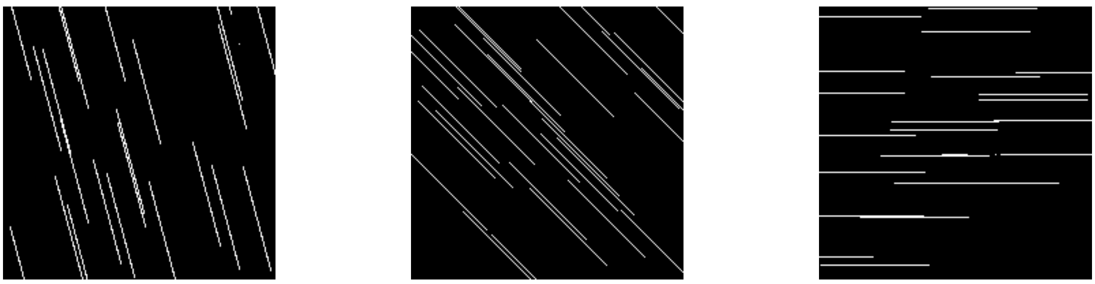
If $d_{\mathrm{pL}}<D/2$ and $d_{\mathrm{pC}}<L/2$, then it is a fiber point and mark it as "1", otherwise it is a matrix point and remains its mark as "0". The line specified by fiber is described as Eq.(12), and then d_{pL} and d_{pC} can be calculated by Eq.(13), (14).

Every time a pixel mark changed to "1", the total fiber volume fraction is calculated. If the volume fraction reaches the predefined value, then terminate the program and export the structure, otherwise keep on judging. While a fiber is generated completely, a new fiber is generated following the same procedure as stated above until the volume fraction meets the predefined value.

Following the method described above, some examples of fibrous structures are generated and shown in Fig. 2 and Fig. 3. During structure generating, fiber diameter and fiber length are keep constant with values 0.005 and 0.4 respectively. It is shown that the generated structures show completely random similar to the real fibrous medium, reflecting the effectiveness of the method to be used to regenerate structure of fibrous material.

(a) ϕ=8% (b) ϕ=10%

Fig. 2 Random distributed fibrous media (RDFM) generated with different fiber volume fraction, D=0.005, L=0.4



(a) θ=15° (b) ϕ=θ=45° (b) ϕ=θ=90°

Fig. 3 Directionally distributed fibrous media (DDFM) generated with different fiber orientation, D=0.005, L=0.4, V=0.04

3.2 Governing equations

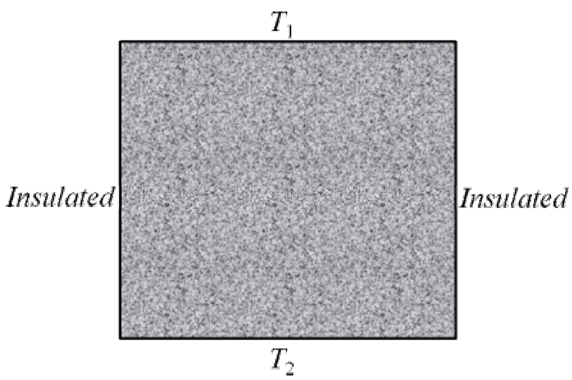


Fig. 4 Schematic of the two dimensional coordinate system

For a two dimensional steady heat transfer problem which combined conduction and radiation without internal heat sources, the energy equation is written as

$$\lambda_i \frac{\partial^2 T}{\partial x^2} + \lambda_i \frac{\partial^2 T}{\partial y^2} - \nabla \bullet \boldsymbol{q}_r = 0 \, , i = m, d \tag{15}$$

where λ is the thermal conductivity, T is temperature, $\nabla \bullet \boldsymbol{q}_r$ is the divergence of radiative heat flux. Subscripts i denotes matrix phase and dispersed phase in multiphase media when it is m, d respectively.

Energy equation can be discretized by using FVM,

$$[(\lambda_i \Delta y \frac{\partial T}{\partial x})_e - (\lambda_i \Delta y \frac{\partial T}{\partial x})_w + (\lambda_i \Delta x \frac{\partial T}{\partial y})_n - (\lambda_i \Delta x \frac{\partial T}{\partial y})_s] - \nabla \bullet \boldsymbol{q}_r \times \Delta V = 0 \tag{16}$$

The temperature gradients on grid interface are calculated by central difference and the thermal conductivities on grid interface are calculated by harmonic average, then Eq.(16) can be rearranged to:

$$T_P = (a_E T_E + a_W T_W + a_N T_N + a_S T_S - \nabla \bullet \boldsymbol{q}_r \times \Delta V) / a_P \tag{17}$$

where $a_E = \lambda_e \Delta y / \Delta x_E$, $a_W = \lambda_w \Delta y / \Delta x_P$, $a_N = \lambda_n \Delta x / \Delta y_N$, $a_S = \lambda_s \Delta x / \Delta y_P$, $a_P = a_E + a_W + a_N + a_S$ 。

In Eq.(17), the divergence of radiative heat flux $\nabla \bullet \boldsymbol{q}_r$ must be first got to calculate the temperature field, it can be expressed as [25],

$$\nabla \bullet \boldsymbol{q}_r = \alpha (4 \sigma_{\mathrm{B}} T^4(\mathbf{r}) - \int_{4\pi} I(\mathbf{r}, \Omega) \mathrm{d}\Omega) \tag{18}$$

where $I(\mathbf{r}, \Omega)$ denotes the radiation intensity, which is a function of position $\mathbf{r}$ and direction Ω. Radiation intensity is determined by solving the RTE,

$$\mu \frac{\mathrm{d}I(\mathbf{r},\Omega)}{\mathrm{d}x} + \xi \frac{\mathrm{d}I(\mathbf{r},\Omega)}{\mathrm{d}y} = -\beta I(\mathbf{r},\Omega) + \alpha I_{\mathrm{b}}(\mathbf{r}) + \frac{\sigma}{4\pi} \int_{4\pi} I(\mathbf{r},\Omega_j) \Phi(\Omega_j,\Omega) \mathrm{d}\Omega_j \tag{19}$$

where μ, ξ is the direction cosine of Ω, β, α, σ are extinction, absorption and scattering coefficient respectively, Φ is the scattering phase function. The left-hand side terms represent the gradient of the intensity in the specified direction, and the right-hand side terms represent the attenuation of intensity by extinction (absorption and scattering), the augmentation of intensity by emitting and the augmentation of intensity by incoming scattering.

Standard RTE is an integro-differential equation, which is difficult to be solved analytically. DOM discretizes the whole space into a finite number of ordinate directions, and uses a set of angular averaged intensity equations over those directions to represent RTE. The integral over incident angular directions is approximated by a weighted sum of the angular quantities, and then the equation of intensity transfer in the m direction is written as

$$\mu^m \frac{\mathrm{d}I^m}{\mathrm{d}x} + \xi^m \frac{\mathrm{d}I^m}{\mathrm{d}y} = -\beta I^m + \alpha I_{\mathrm{b}} + \frac{\sigma}{4\pi} [\sum_{l=1}^{N\Omega} w^l I^l \Phi^{m,l}] \tag{20}$$

In equations, m and l denote outgoing and incoming directions, respectively. Integrating Eq.(20) over the control volume of $dx{\cdot}dy$ gives

$$\mu^m A_x(I_e^m - I_w^m) + \xi^m A_y(I_n^m - I_s^m) = -\beta I_P^m \Delta V + \alpha I_{\mathrm{bk},P}\Delta V + \frac{\sigma}{4\pi}[\sum_{l=1}^{N\Omega} w^l I_P^l \Phi^{m,l}]\Delta V \quad (21)$$

Subscripts *e,w,n,s* denote the intensity on interface and *E,W,N,S,P* denote the intensity on node. ΔV is the control volume. Assuming intensity on interface and node has the relationship as follows:

$$I_P^m = f_x I_e^m + (1-f_x) I_w^m = f_y I_n^m + (1-f_y) I_s^m \quad (22)$$

f_x and f_y are weighting factors. Substituting Eq.(22) into Eq.(21) gives

$$I_P^m = \frac{\mu^m A_x f_y I_w^m + \xi^m A_y f_x I_s^m + S_P^m f_x f_y \Delta V}{\mu^m A_x f_y + \xi^m A_y f_x + D_P^m f_x f_y \Delta V} \quad (23)$$

In Eq.(23), $D_P^m = \beta - \frac{\sigma}{4\pi} w^l \Phi^{m,m}$, $S_{k,P}^m = \alpha I_{\mathrm{b},P} + \frac{\sigma}{4\pi} \sum_{l,l\neq m} w^l I_P^l \Phi^{m,l}$

Eq. (23) has been written for a direction with positive direction cosine, which is an intensity moving forward and to the right, entering through A_S and A_W faces and exits through A_N and A_E. It should be solved forwardly. In this study, weighting factors use a value of 0.5 which yield the “diamond difference” relations and boundary conditions of four faces are all diffuse-black which are as follows,

$$I_w(\boldsymbol{s}) = \varepsilon_w I_{\mathrm{b},w} + \frac{1-\varepsilon_w}{\pi} \int_{\boldsymbol{n}_w \bullet \boldsymbol{s}_i < 0} I_w(\boldsymbol{s}_i) |\boldsymbol{n}_w \bullet \boldsymbol{s}_i| \mathrm{d}\Omega_{\mathrm{i}} \quad (24)$$

where ε_{w} is the wall emissivity and $\boldsymbol{n}_{\mathrm{w}}$ is the normal vector of boundary.

The discrete form of boundary condition (Eq.(25)) is also given,

$$I_w^m = \varepsilon_w \frac{\sigma T_w^4}{\pi} + \frac{1-\varepsilon_w}{\pi} \sum_{\boldsymbol{n}_w \bullet \boldsymbol{s}_l < 0} w^l I_w^l |\boldsymbol{n}_w \bullet \boldsymbol{s}_l| \quad (\boldsymbol{n}_w \bullet \boldsymbol{s}_m > 0) \quad (25)$$

DOM is used to calculate the radiation intensity to get the divergence of radiative heat flux, then substituting the divergence of radiative heat flux into Eq. (17) to recalculate the temperature field. The new temperature will be used to recalculate the radiation intensity again. This iterative process should be continued until the temperature field and radiation intensity field both get convergent.

When convergent temperature field and radiation intensity field is got, heat flux in Y direction at each node is calculated by,

$$Q(i,j)_y = -k\frac{\partial T}{\partial x} + \sum_m^M w_m \mu_m I^m \quad (26)$$

where the right hand terms are conduction heat flux and radiation heat flux respectively. Total heat flux in Y direction is as follows,

$$Q_y = \sum_{i=1}^{L_1} Q(i,j)_y \quad (27)$$

where L_1 is the node number in X direction. Then effective thermal conductivity accounting for both heat conduction and thermal radiation is calculated according to

$$\lambda_{\mathrm{e}} = \frac{Q_y \times XL}{\Delta T} \quad (28)$$

XL and ΔT are thickness and temperature difference between the top and bottom face.

3.3 Validation

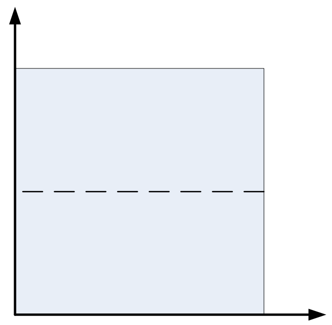


Fig. 5 Schematic of the coordinate system in the rectangular enclosure

As validation study, we consider coupled conduction and radiation heat transfer in a 2-D rectangular enclosure in which a nonscattering medium is included (Fig. 5). The problem has been numerically studied by Kim [33]. Boundary temperature is as follows: T_1=300 K, T_2= T_3= T_4=150 K, and all of the boundary walls are black with emissivities equal 1.0. The length and width of enclosure is 1.0, and extinction coefficient of medium is 1.0. Thermal conductivity equals 8.3916 (W/m·K) and 0.83916 (W/m·K) which correspond to the conduction-radiation parameter $N=\beta\lambda/(4\sigma T_1^3)$ equals 1 and 0.1 respectively.

Fig. 6 and Fig. 7 give comparison of dimensionless temperature and total heat flux between the present study and Kim's. It is seen from both figures that the temperature and heat flux agree well with that reported in the literature.

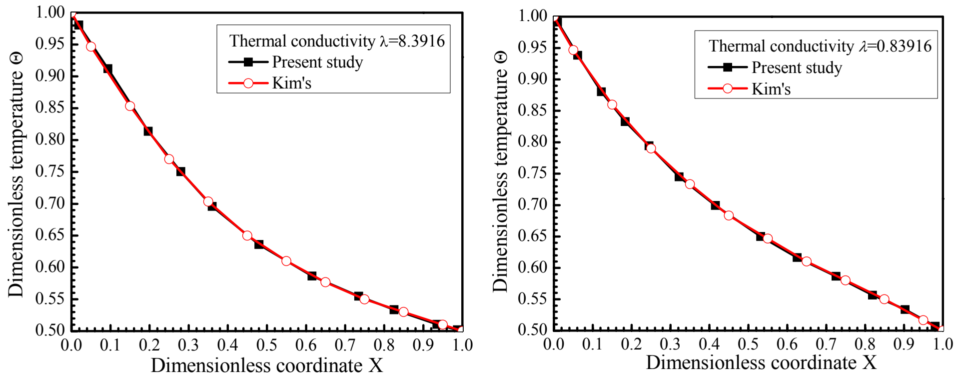


Fig. 6 Dimensionless temperature along the symmetry line Y=0.5 for two thermal conductivities

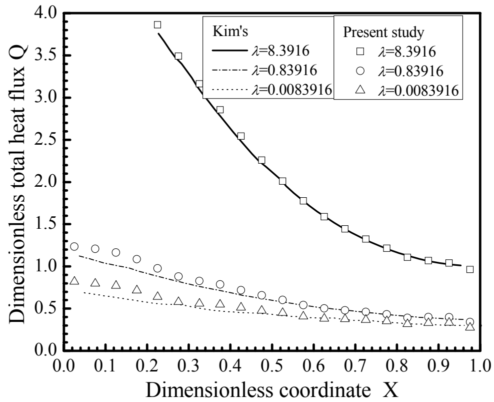


Fig. 7 Dimensionless total heat flux along the symmetry line Y=0.5 for two thermal conductivities

4. Parameter identification

After the model has been developed, thermal properties should be identified to study the heat transfer. This includes the temperature-dependent thermal conductivities of fiber and matrix media as well as the material's radiative properties, such as each phase's spectral extinction coefficient and scattering albedo of the media.

In this paper, we will take a kind of insulation material as illustration. Fiber is assumed to be silica fibers, and matrix material is selected as silica aerogel. Their thermal conductivities are all determined from reference [41] and listed in Table 2.

Table 2 Temperature-dependent thermal conductivities of fiber and silica aerogel

Temperature K	λ_f W/m·K	λ_m W/m·K	Temperature K	λ_f W/m·K	λ_m W/m·K
300	1.38	0.0127	900	2.47	0.0228
400	1.51	0.0139	1000	2.88	0.0264
500	1.63	0.0150	1100	3.41	0.0309
600	1.76	0.0161	1200	4.08	0.0368
700	1.94	0.0177	1300	4.92	0.0444
800	2.17	0.0200	1400	5.94	0.0571

Considering the radiative properties, we will use datum given in reference [42] to be the spectral extinction coefficient of silica aerogel $\sigma_{e\lambda,gel,}$, and apply Mie theory to calculate the spectral extinction coefficient of fibers $\sigma_{e\lambda,f}$.

In fibrous materials, fiber could be seemed as infinite cylinder when the ratio of its length and diameter is very large, thus Mie theory could be applied to get the radiative properties of fiber. According to Mie theory, scattering and extinction factor of an infinite cylinder can be calculated by the following equations [43]:

$$Q_{s\lambda,I}(\alpha) = \frac{2}{\chi}\{|b_{0I}|^2 + 2\sum_{n=1}^{\infty}[|b_{nI}|^2 + 2|a_{nI}|^2]\}$$

$$Q_{e\lambda,I}(\alpha) = \frac{2}{\chi}\mathrm{Re}(b_{0I} + 2\sum_{n=1}^{\infty} b_{nI})$$

$$Q_{s\lambda,II}(\alpha) = \frac{2}{\chi}\{|a_{0II}|^2 + 2\sum_{n=1}^{\infty}[|a_{nII}|^2 + |b_{nII}|^2]\}$$

$$Q_{e\lambda,II}(\alpha) = \frac{2}{\chi}\mathrm{Re}(a_{0II} + 2\sum_{n=1}^{\infty} a_{nII}) = \frac{2}{\chi}\mathrm{Re}(T_2(\Theta = 0^\circ))$$

$$Q_{s\lambda} = \frac{1}{2}(Q_{s\lambda,I} + Q_{s\lambda,II}),\ Q_{e\lambda} = \frac{1}{2}(Q_{e\lambda,I} + Q_{e\lambda,II}) \qquad (29)$$

Re denotes the real part of a complex number, r is the particle radius, χ is the size factor and is defined as $\pi D/\lambda$, where $D=2r$ is the particle diameter, $Q_{s\lambda}$ and $Q_{e\lambda}$ are the spectral scattering and extinction factor. a_{nI}, b_{nI}, a_{nII}, b_{nII} are Mie coefficients and they are functions of fiber's complex refractive index (n-ki). Their detailed expressions can refer to reference [43]. In this study, the complex refractive indexes needed for calculating the scattering and extinction factors are selected from reference [44].

Once single fiber radiative properties and its size and orientation distributing functions are fixed, total radiative properties of fibers can be calculated by integrating each fiber's. Cunnington and Lee [45] proposed Eq.(30) to calculate the total scattering, absorbing and extinction coefficients of fibers related with single fiber's scattering, absorption and extinction factors.

$$\{\bar{\sigma}_{e\lambda},\bar{\sigma}_{s\lambda},\bar{\sigma}_{a\lambda}\}=\int_{\omega_{f1}}^{\omega_{f2}}\int_{\xi_{f1}}^{\xi_{f2}}\int_0^{\infty}2r\{Q_{e\lambda},Q_{s\lambda},Q_{a\lambda}\}N(r(\mathbf{R}_f))drd^2F \tag{30}$$

In the above equation, d^2F and $N(r(\mathbf{R}_f))$ are the orientation distribution and number size distribution; the limits of integration (ξ_{f1}, ξ_{f2}, ω_{f1}, ω_{f2}) denote the range of angular orientation of the fibers.

If fibers are randomly oriented in space, the radiation coefficients will be independent of angle and can be calculated as follows [44],

$$\{\bar{\sigma}_{e\lambda},\bar{\sigma}_{s\lambda},\bar{\sigma}_{a\lambda}\}=\frac{2f_v}{\pi r}\int_0^{\pi/2}\{Q_{e\lambda}(\theta),Q_{s\lambda}(\theta),Q_{a\lambda}(\theta)\}\cos\theta\mathrm{d}\theta \tag{31}$$

If all the fibers are oriented in a specified orientation and with the same radius, then Eq. (30) becomes

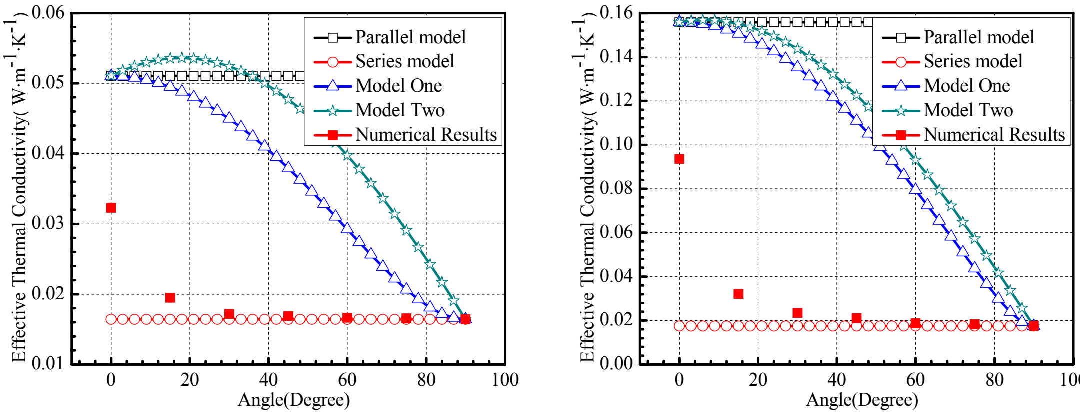
$$\{\bar{\sigma}_{e\lambda},\bar{\sigma}_{s\lambda},\bar{\sigma}_{a\lambda}\}=\int_{\omega_{f1}}^{\omega_{f2}}\int_{\xi_{f1}}^{\xi_{f2}}\int_0^{\infty}2r\{Q_{e\lambda},Q_{s\lambda},Q_{a\lambda}\}N(r(\mathbf{R}_f))drd^2F=\frac{2f_v}{\pi r}\{Q_{e\lambda},Q_{s\lambda},Q_{a\lambda}\} \tag{32}$$

5. Results and discussion

The preceding sections show the theoretical model and numerical method for calaulating the effective thermal conductivity of fibrous media. In this section, effective thermal conductivity of fibrous media which accounts for heat conduction only (CTC) or for combined conductive and radiative heat transfer (CRTC) are both considered. Theoretical and numerical methods are used and results calculated by two methods are compared to each other.

5.1 Conductive thermal conductivity (CTC)

5.1.1 Directionally distributed fibrous media(DDFM)



(a) V=2% (b) V=8%

Fig. 8 Comparison of effective thermal conductivity between numerical results and several theoretical results

Fig. 8 gives comparison of effective thermal conductivity between the numerical calculating results and several theoretical results. Fiber volume fractions are 2% and 8% respectively, while other parameters are selected as Table 3 shows. Because of the independence with angle contained by fiber axis and heat flux, the results that calculated by Parallel model and Series model exhibit horizontal lines which do not change with angle. Model One and Model Two indicate that effective thermal conductivity decreases with angle. This is generally true, but it is found that both the variation trend and the exactly predicted value do not agree with numerical result. Besides fiber orientation, fiber length and thermal conductivity ratio of fiber and matrix could also influence the effective thermal conductivity, so it is necessary to develop a model that takes these factors into account.

Table 3 Thermal conductivities and structure parameters used for Fig. 8

Parameter	Value	Parameter	Value
λ_f	1.762717	D	0.005
λ_m	0.016105	L	0.4

According to the numerical results, effective thermal conductivity looks like decreasing with the angle exponentially. So first order exponentially decay model is used as the base model to be fitted. Meanwhile, fitted model should also satisfy the following conditions that as angle equals 90 degree, effective thermal conductivity equals to the value that calculated by Series model. Based on the reasons that stated above, following model is proposed,

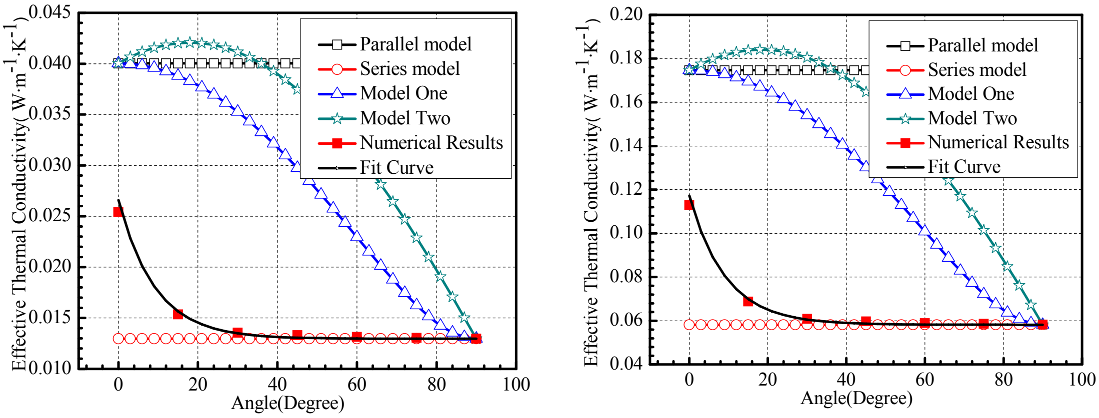
$$\lambda_{\text{eff}} = A(\phi, \lambda_{\text{f}} / \lambda_m, L) * (\lambda_{\text{parallel}} - \lambda_{\text{series}}) * \exp(-\theta / B(\phi, \lambda_{\text{f}} / \lambda_{\text{m}}, L)) + C(\phi, \lambda_{\text{f}} / \lambda_{\text{m}}, L)\lambda_{\text{series}} \tag{33}$$

In order to determine parameters A, B and C, several cases are executed, and it seems that parameter C is approximately unit and independent with other contributing factors. While parameter B is related to the thermal conductivity ratio of fiber and matrix. Parameter A can be affected both by fiber length and thermal conductivity ratio. Based on large amount calculation, the modified model is finally determined as

$$\lambda_{\text{eff}} = (2.087 - 1.2544 e^{-1/0.07646\alpha})(1 - e^{-L/0.55})(\lambda_{\text{parallel}} - \lambda_{\text{series}}) e^{-\theta/(8.17 + 120.13/\alpha - 115.32/\alpha^2)} + \lambda_{\text{series}} \tag{34}$$

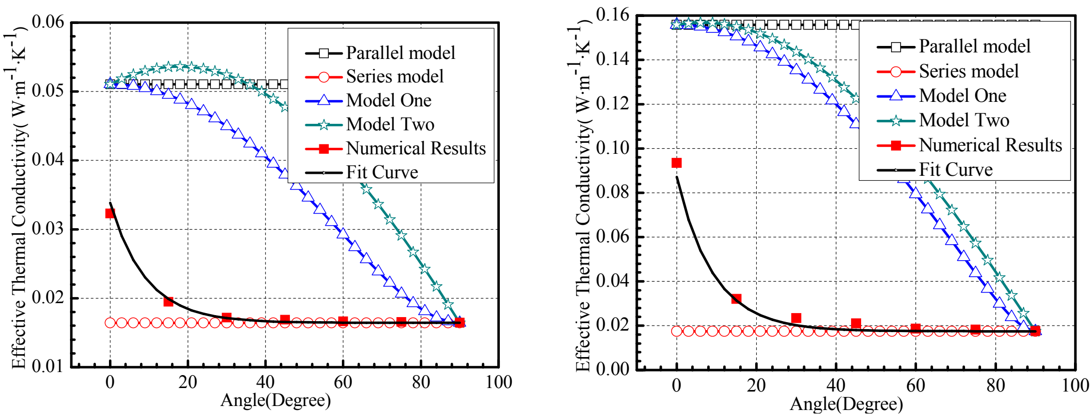
In Eq.(34), α denotes the thermal conductivity ratio of fiber phase and matrix phase, L is the fiber length which is a relative ratio compared with material size, θ is the angle that contained by fiber axis and heat flux direction. $\lambda_{\text{parallel}}$ and λ_{series} are effective thermal conductivity that calculated by Parallel model and Series model respectively.

While the modified model is established, several cases with different physical properties and structure parameters are investigated. The results are compared with the modified model and traditional models as Fig. 9 ~ Fig. 12 show. It is obviously that modified model agree well with all of the different cases which indicate that modified model can be effectively used to predict the effective thermal conductivity of fibrous media.



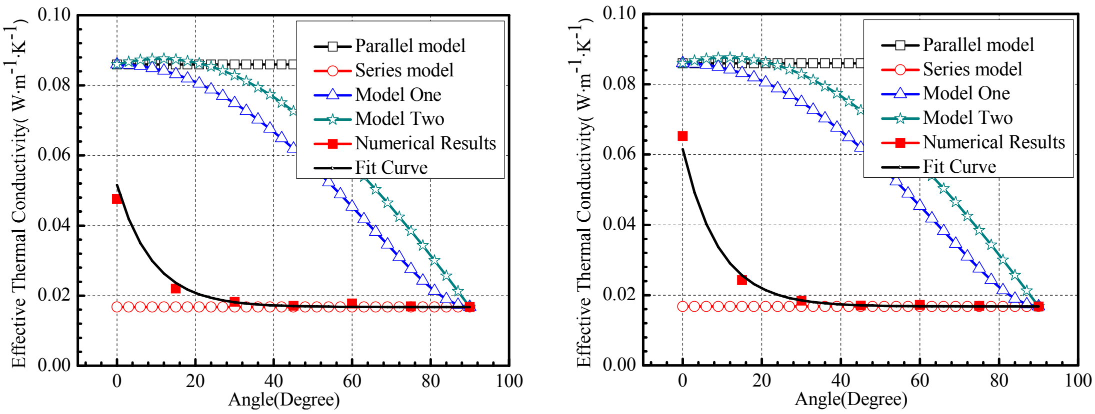
(a) T=300 K (b) T=1400 K

Fig. 9 Variation of effective thermal conductivity with angle under different temperature, ϕ=2%, L=0.4, D=0.005 (a) λ_m=0.0127, λ_f= 1.37938 (b) λ_m=0.05705797, λ_f=5.939028



(a) ϕ=2% (b) ϕ=8%

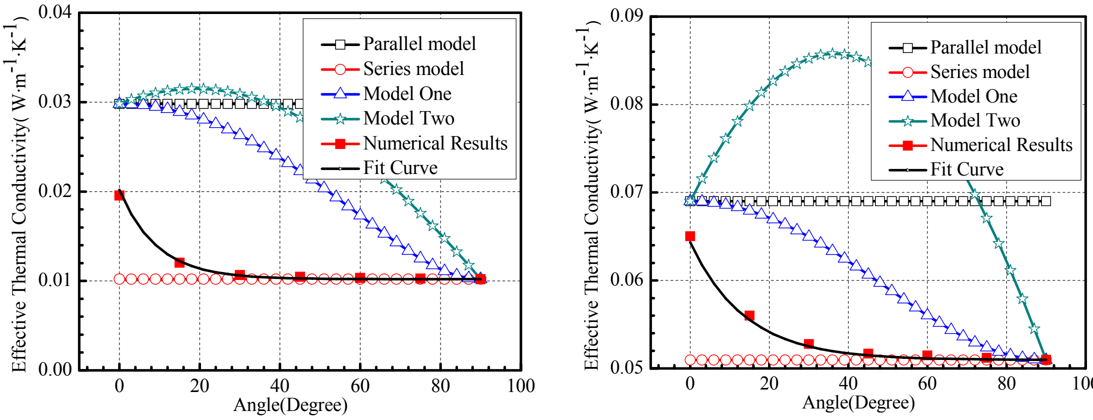
Fig. 10 Variation of effective thermal conductivity with angle under different fiber volume fraction, L=0.4, D=0.005, λ_m=0.01610507, λ_f=1.762717 (a) ϕ=2% (b) ϕ=8%



(a) L=0.4 (b) L=0.6

Fig. 11 Variation of effective thermal conductivity with angle under different fiber length, ϕ=4%,

D=0.005, λ_m=0.01610507, λ_f=1.762717 (a) L=0.4 (b) L=0.6

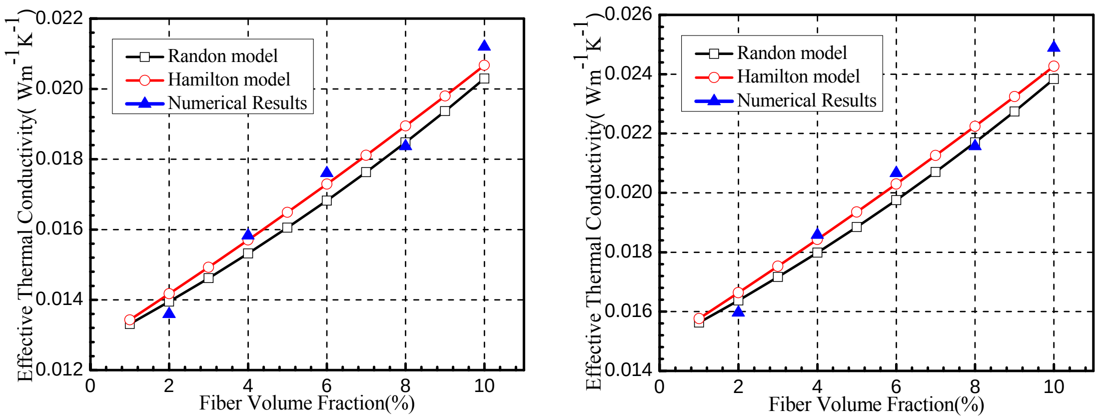


(a) α=0.01 (b) α=0.05

Fig. 12 Variation of effective thermal conductivity with angle under different thermal conductivity ratio, ϕ=2%, L=0.4, D=0.005 (a) α=0.01 (b) L=α=0.05

5.1.2 Random distributed fibrous media (RDFM)

For fibrous media which has randomly distributed fibers, we have also numerically computed their effective thermal conductivity and compared with theoretical models that usually be used. The results are shown in Fig. 13. It can be seen from the figure that numerical results agreed well with Random model and Hamilton model, which indicates the effectiveness of these two models for predicting fibrous media's effective thermal conductivity.



(a) λ_m=0.0127, λ_f=1.38, L=0.4, D=0.005 (b) λ_m=0.0149, λ_f=1.63, L=0.4, D=0.005

Fig. 13 Comparison of numerical results and theoretical models for randomly distributed fibrous media

5.2 Combined conductive and radiative thermal conductivity (CRTC)

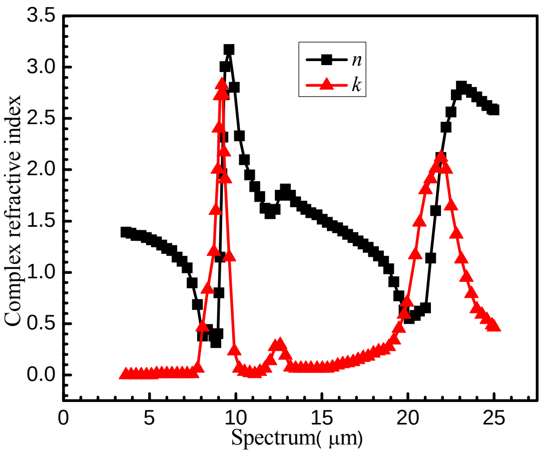


Fig. 14 Complex refractive index of silica fiber

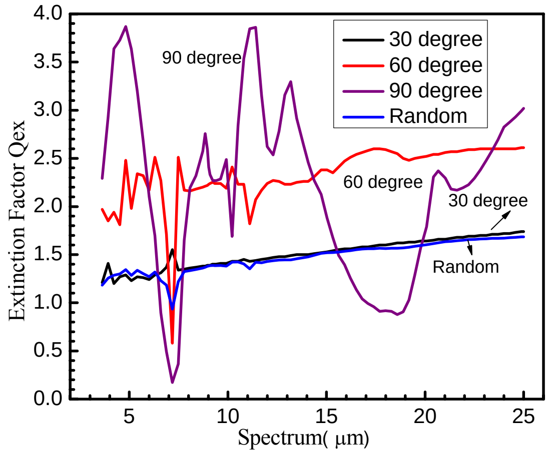


Fig. 15 Spectral extinction factor with different angle and randomly distributed, calculated according to Mie theory

Insulation material always works in high temperature environment. This makes thermal radiation in participating media becomes significantly, leading to a high increase of heat transfer. In order to investigate the radiation heat transfer, spectral extinction coefficients of fibers and silica aerogel should be got firstly. Silica aerogel's spectral extinction coefficient was selected from reference [42], and fiber's was calculated according to Mie theory. Fig. 14 gives the complex refractive index of silica fiber and Fig. 15 shows the spectral extinction factors for fibers with different angle contained between fiber axis and heat flux direction as well as randomly distributed fibers. The fibers were assumed to be infinite cylinders and with 8μm diameter. The spectral extinction factors shown in Fig. 15 will be used as the base in the coming sections when calculating the spectral extinction coefficient and radiative conductivity.

5.2.1 CRTC with different fiber volume fraction

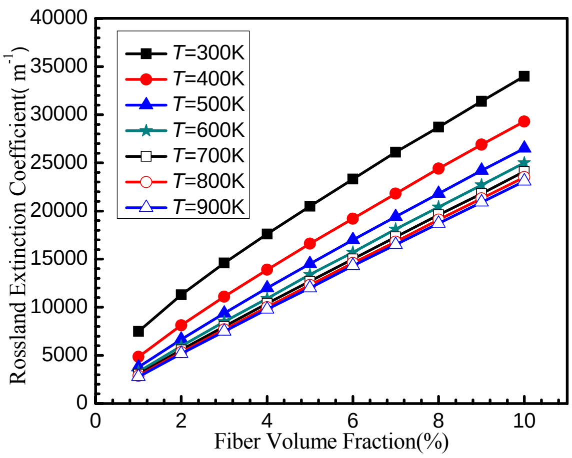


Fig. 16 Rossland extinction coefficients with different fiber volume fraction at different temperature

For RDFM, the relationship between spectral extinction factors and spectral extinction coefficient are as Eq.(31) shows. Under different fiber volume fraction, the spectral extinction coefficients and absorption coefficients were calculated, and then Rossland average were made on spectral extinction coefficients for different temperature. This gives the Rossland mean extinction coefficients as Fig. 16 shows.

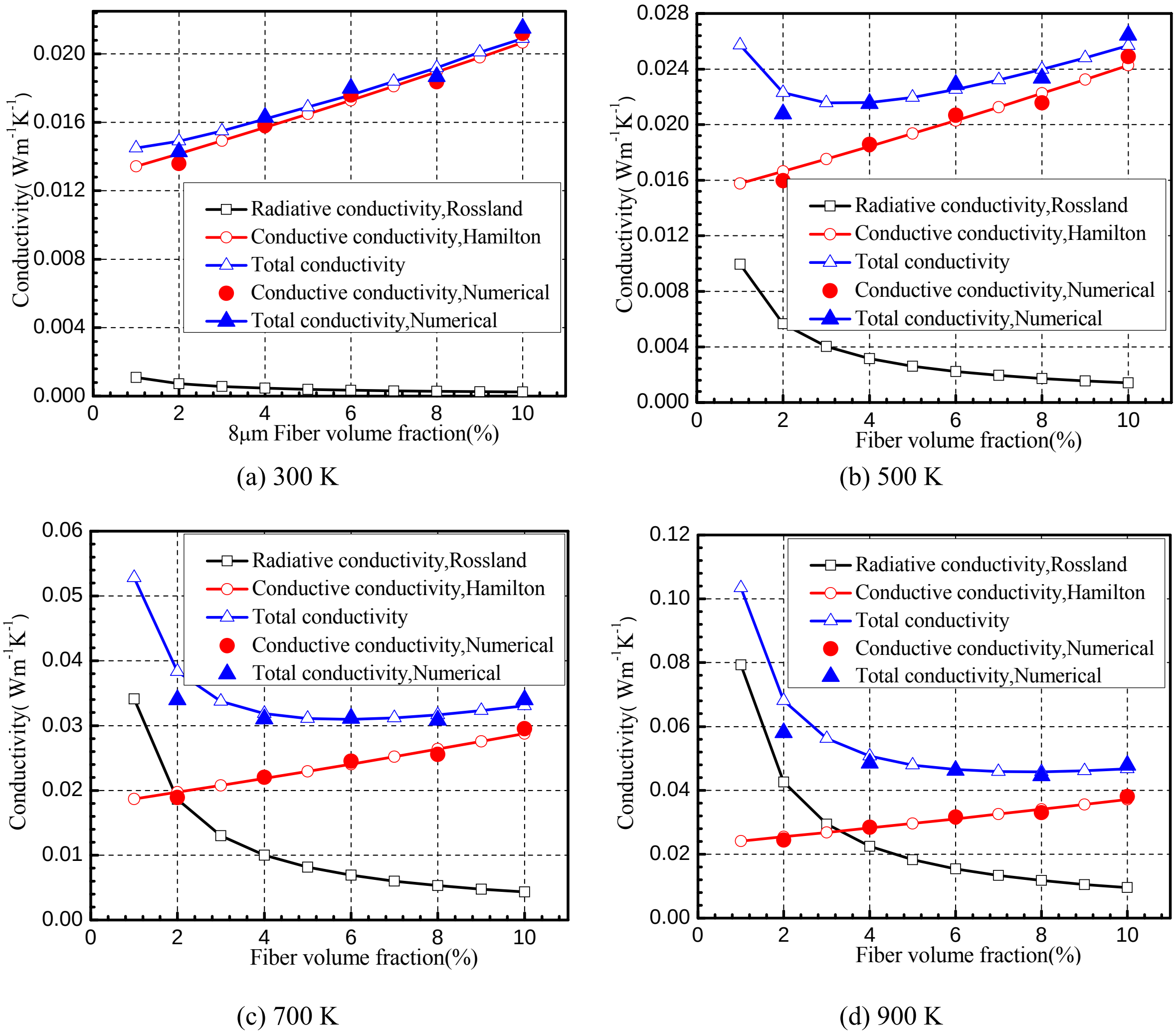


Fig. 17 Comparison of numerical results and theoretical results of CRTC of fibrous media with different volume fraction

Fig. 17 shows variation of RTC, CTC and total conductivity (CRTC) of composite fibrous media with different fiber volume fraction, (a) (b) (c) (e) correspond to situations with 8μm fibers at 300 K, 500 K, 700 K and 900 K respectively. The volume fraction changed between 0~10% and fibers were assumed to be distributed randomly in the space.

As shown in Fig. 17, when temperature is low, radiative heat transfer is very small and heat conduction is the main heat transfer mode. While the temperature increases, radiation increases obviously and becomes the main heat transfer mode. The figures also shown that as fiber volume fraction increased, RTC decreased greatly which meant that radiative heat transfer is restrained well by the fibers.

5.2.2 CRTC with different fiber orientation

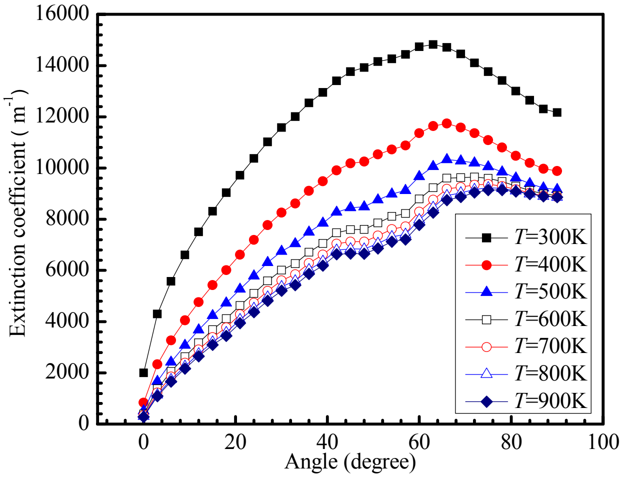
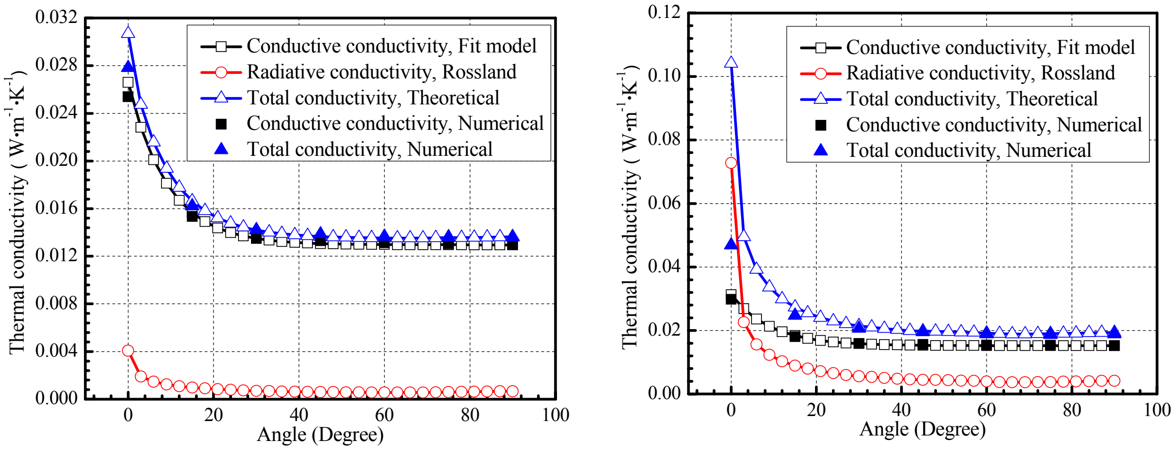


Fig. 18 Rossland extinction coefficients with different angle at different temperature



(a) 300 K (b) 500 K

Fig. 19 Comparison of numerical results and theoretical results of CRTC of fibrous media with different angle

For fibrous media with specified orientation, its spectral extinction coefficient was calculated according to Eq.(32). Fig. 18 shows the Rossland extinction coefficient of silica aerogel composite with directionally distributed fibers under different temperature. Fig. 19 gives the comparison of thermal conductivities that were calculated by numerical method and theoretical method. It can be seen that CTC decreases with the angle, while RTC decreases with the angle firstly and then increases slightly when angle approaches to 90°. Numerical results and theoretical results agreed well with each other except the situation that angle is around 0°. This is because that as the angle close to 0°, extinction coefficient of silica aerogel composite with fiber gets very small (see Fig. 18), which makes Rossland approximation invalid for RTC. And the CRTC calculated based on Rossland approximation deviates greatly with the results that calculated by numerical method. Fig. 19 shows that fiber orientation influences greatly both on heat conduction and thermal radiation, this should be considered while the optimization of insulating performance of fibrous media.

6. Conclusion

In this work, the theoretically and numerically investigation of the radiation and conduction heat transfer in porous fibrous material were conducted. For theoretical study, several models and Rossland approximation were used to calculate the effective thermal conductivity of two types of fibrous materials. And for numerical computation, the FVM coupling the DOM was used to study the conductive and radiative heat transfer. Mie theory was employed to obtain the radiative properties which are needed for radiative heat transfer, and a kind of structure generation method was applied to simulate fibrous media structure which is used for numerical study. With the coupled modeling, the mainly conclusions were drawn as below:

1. The results calculated by Random model and Hamilton model for CTC of randomly distributed fibrous media agreed well with the results that calculated by numerical method, indicating that these two models can work well for predicting RDFM's effective thermal conductivity.

2. CTC of DDFM decays exponentially with angle between the heat flux direction and fiber orientation. This cannot be reflected by the existing models. A modified model is proposed for DDFM which shows accurate predicting ability for conductive thermal conductivity.

3. For isotropic media, besides cases where material's optical thick is very small, the effective thermal conductivity values that predicted by theoretical models combined with Rossland approximation and numerical method which is FVM combined with DOM can agree well with each other. For materials with small optical thick, Rossland approximation is invalid and the predicted value is far away from the numerical results.

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